Cohort mortality risk or adverse selection in annuity markets?

By

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July 2016

Forthcoming *Journal of Public Economics*

Abstract

We show that tests for adverse selection in annuity markets using prices are not identified. Within the UK annuity market, different annuity products create the potential for a Rothschild-Stiglitz separating equilibrium as different risk types could choose different annuities. Empirical analyses using the "money's worth" suggest that prices are indeed consistent with this explanation. However, we show that this pattern of annuity prices would also result from the actions of regulated annuity providers who must reserve against cohort mortality risk. Annuity products that might attract different consumer risk types also have different risks for the provider.

Keywords: Adverse selection, insurance markets, annuities

JEL codes: G22, D4, D82

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This article has benefited from presentations at Universities of Bath, Bristol, Edinburgh, McMaster, Queen Mary, NHH (Bergen); Paris Dauphine, Sao Paulo, York (Toronto) and Zhejiang (Hangzhou); South West British Accounting and Finance Conference, Aberystwyth, 2012; Annuities Workshop at Eidgenössische Technische Hochschule, Zürich, 2011; Royal Economic Society Conference, 2013; British Accounting and Finance Association Conference, 2013, Allianz-Oxford Pensions Conference, September 2013, Netspar Conference, Venice, June 2014, Money-Macro-Finance Conference, Cardiff, September 2015, and comments from Tom Boardman, Antoine Bommier, Narat Charupat, David DeMeza, Con Keating, Moshe Milevsky, James Poterba and David Webb. The authors gratefully acknowledge support for this research from NETSPAR.

1. Introduction

Ever since the development of the theoretical model of Rothschild and Stiglitz (1976) focusing on the rôle of asymmetric information in insurance markets, the search for empirical evidence on adverse selection has yielded conflicting findings depending on the characteristics of the particular market (Cohen and Siegelman, 2010). A common approach has been to investigate the positive correlation property (PCP), whereby higher-risk individuals buy more insurance. In the context of life annuities, higher risk corresponds to higher life expectancy and a direct test would be that individuals who have private information about their life expectancy select into back-loaded annuity products and hence individuals who buy back-loaded products live longer. However, PCP tests are not generally feasible because individuals' purchases of annuities are proprietary information and are not widely available. An alternative test of the same phenomenon is whether annuity providers (life insurers) recognise adverse selection and price accordingly, leading to different money's worths for different annuity products. Mitchell et al (1999) and Finkelstein and Poterba (2002) have examined the pricing of life annuities using the money's worth (MW) metric, defined as the ratio of the expected value of annuity payments to the premium paid.[1] This empirical literature generally suggest that: (i) the MW is less than one; and (ii) the MW of back-loaded annuities (such as escalating or real annuities where the expected duration is longer) is less than that for level annuities. For example, Finkelstein and Poterba (F&P) (2002, p.46) reports that the money's worth of level annuities for 65-year old males is 0.900, but for escalating annuities is 0.856. These two observations have been interpreted as evidence of adverse selection, that annuitants have more information about their life expectancy than insurance companies, which is then reflected in equilibrium annuity prices.

In this paper we evaluate the identifying assumptions used to test for adverse selection through analysis of prices in annuity markets. We demonstrate that these same facts would also be consistent with a model where there is no adverse selection

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[1] James and Song (2001) and Cannon and Tonks (2008) provide an international comparison of money's worth studies across a wide range of countries. Since then further analyses have been conducted for Canada (Milevsky and Shao, 2011); for Germany (Kaschützke and Maurer, 2011); for the Netherlands (Cannon, Stevens and Tonks, 2012); for Singapore (Fong, Mitchell and Koh, 2011); and for Switzerland (Bütler and Staubli, 2011).

and where the variation in annuity rates for different types of annuity were due to the different costs of supplying annuities. The providers of annuity contracts are exposed to the survival or conversely mortality risks of annuitants. Either because life insurers are prudent or because of regulatory requirements, riskier liabilities such as escalating or real annuities have to be priced to ensure sufficient reserves are available and matched to similar assets and these effects make them more costly. Although idiosyncratic mortality risks are diversified in a large pool of annuitants, a life insurer still faces the risk from predicting cohort mortality over a long period. The route by which this cohort risk and adverse selection affect annuity prices is the same, namely the duration of the annuity. This makes identifying the importance of the two explanations for annuity prices difficult. In this paper we quantify the costs of these cohort mortality risks and show they are sufficiently large to explain much of the observed variations in the money's worth, leaving a smaller rôle for adverse selection.

The paper is structured as follows. In the next section we discuss the organisation of annuity markets in the UK, review the theory and evidence for adverse selection and discuss the consequences of mortality risk for annuity pricing. In section 3 we describe the conventional MW measure, examine the implications of adverse selection and prudential reserving for the money's worth, summarise the data used in our empirical analysis, and provide time series calculations of the MW by annuitant age and product type. In section 4 we show how a probability distribution of the value of an annuity can be constructed from a stochastic mortality model. We use this to measure the risk for annuities and the consequences when a researcher calculates the MW based on a deterministic projection of mortality, but when annuity providers are pricing to take into account the financial risk associated with mortality risk and a given set of interest rates. Section 5 concludes.

2. Adverse Selection and Annuity Markets

In this section we describe the structure and regulation of UK annuity markets and explain how the theory of adverse selection developed for general insurance relates to the specific characteristics of annuities markets. We summarise the results of MW and PCP tests and discuss whether they constitute evidence for adverse selection. We then discuss the need for life insurers to reserve against long-run cohort risk that cannot be diversified away by pooling a large number of annuitants and the consequences of this for the pricing of annuities.

We first describe the structure of the UK's annuity markets. Due to the compulsory annuitisation of wealth accumulated in tax-efficient defined contribution personal pension schemes up until 2014, the UK annuity market was the largest in the world, accounting for almost half of all annuities sold worldwide (worth £11 billion per year; HM Treasury, 2010).[2] A variety of annuity types were allowed by the tax authorities so, in principle, life insurers could price annuities to separate different risk types as described in the Rothschild-Stiglitz (1976) model (henceforth RS), and extended to the annuity market by Eckstein, Eichenbaum and Peled (1985). The RS model assumes that the insurer can observe the quantity of insurance purchased, but this is not a valid assumption in the annuity market. Anyone purchasing an annuity would almost certainly have some additional annuitised wealth (through pensions or annuities purchased from other providers) and non-annuitised wealth (both financial and housing assets), none of which are observable by the life insurer. Even if a life insurer did observe the proportion of wealth annuitised, an annuitant could still choose to consume less than their annuity income initially and save in a non-annuity product and this could undermine the ability of life insurers to separate different risk types. Finkelstein, Poterba and Rothschild (2008, Figure 4) illustrate numerical simulations showing that to separate individuals it would be necessary to offer an annuity product to short-lived individuals where payments in the distant future (above age 90) were negligible and no such annuity types are observed in practice. In fact, Abel (1985) and Walliser (2000) show that the combination of unobservable quantities and adverse selection can result in a pooling equilibrium. We conclude that the theoretical literature on annuities is ambivalent on whether adverse selection will be characterised by a separating or pooling equilibrium.

[2] Despite some exemptions and changes to the rules during this period, most individual DC pension scheme participants had to annuitise 75 per cent of personal pension wealth accumulated in a tax-exempt savings vehicle by age 75. This requirement was removed in March 2014.

Turning to the empirical evidence, F&P (2002) demonstrate that the pattern of UK annuity prices for different product types is consistent with adverse selection. There are two general product types that might signal life expectancy: first, annuities where the first five years' payments are "guaranteed" (i.e. not life contingent) have less insurance than simple annuities and are more valuable to a short-lived annuitant who values bequests ("front-loaded" annuities). Second, annuities whose payments are escalating in nominal terms, or indexed to inflation are "back loaded" and should be more valuable to longer-lived annuitants. F&P (2002) show that the MW of back-loaded annuities are lower than the MW of front-loaded annuities, consistent with the predictions of a separating equilibrium: henceforth we refer to this as the *price test*. This price test computes the expected present value of an annuity stream, and relies upon using projected mortalities. Either implicitly or explicitly these are uncertain forecasts raising the question of how to incorporate forecast uncertainty explicitly into the evaluation of annuity prices.

An alternative test for establishing the presence of adverse selection, is to check the positive-correlation property (Chiappori et al, 2006), since higher-risk (i.e. longer-lived) individuals should purchase more longevity insurance. Using data on individual policies from a life insurer for 1980-98, F&P (2004) show that annuitants who purchase an annuity with a guarantee period tend to be shorter-lived and those who buy an escalating or real annuity are longer lived, consistent with the RS separating equilibrium. Using policies from another company for 1988-94, Einav, Finkelstein and Schrimpf (2010) find less conclusive evidence, since annuitants who purchase an annuity with a ten-year guarantee are longer lived than those with a five-year guarantee (and in some cases longer-lived than those with no guarantee).

PCP tests for asymmetric information have been criticised by De Meza and Webb (2014) who argue that, under the standard assumptions of actuarially fair pricing and identical preferences, the availability of contracts with different insurance coverage implies the existence of asymmetric information. This is because under symmetric information all risk-averse individuals would choose full cover: the presence of multiple contracts only shows that at least some of the standard assumptions are invalid, not that there is asymmetric information. In the context of accident insurance, De Meza and Webb propose modifying the standard set of assumptions to allow for differential claims-processing costs across contracts, with claims costs non-increasing in the level of insurance cover (for example, due to fixed costs). The inclusion of such costs can then generate multiple cover levels under symmetric information (where individuals will choose their level of cover taking into account expected claims costs). When there is asymmetric information, a non-zero correlation (between risk-types and cover) does not imply asymmetric information; nor does a zero correlation exclude asymmetric information. However, this issue of claims-processing costs cannot be simply translated to the life-annuity scenario, since life annuities pay a stream of payments dependent on life length and there is no direct analogue of a claim or associated costs.

A final issue is that annuity choice may be affected by behavioural issues. A typical annuitant might only be expected to purchase one annuity and there is no scope for learning about the product through experience: so annuity purchase is a plausible scenario for decisions to be affected by framing effects (Benartzi et al, 2011; Beshears et al, 2013). Indeed, the finding in Einav et al. (2010) that the vast majority (87 per cent) of annuities had a five-year guarantee (the "middle option") suggests that choice of annuity type is due to institutional or behavioural factors and so selection effects are not just due to asymmetric information.

We now turn to how cohort mortality risk affects annuity pricing. Compared to other forms of insurance, the cost of a providing an annuity is peculiarly difficult to estimate because of the long-term nature of the product: a 65-year old purchasing an annuity might live for another forty years. This means that estimates of costs must be based upon very long-term projections and introduces an element of uncertainty for the insurer that is less important in general insurance. Given that the uncertainty of mortality forecasts increases with the time horizon, it also follows that annuities with a longer duration are also higher risk to the annuity provider, suggesting that they may need to offer a lower annuity rate if the annuity provider is risk averse or facing regulatory constraints, and this would automatically result in a lower MW for back-loaded products.

Annuities in the UK are sold by life insurers whose liabilities consist of future annuity payments, and whose assets are predominantly high-quality bonds.[3] Annuities payments defined in nominal terms can be matched with conventional bonds, and those defined in real terms matched to inflation-linked bonds,[4] so the risk to forecasting cohort mortality constitutes the largest component of the total risk to selling an annuity. Because individual annuitants lack either adequate incentives or the ability to monitor the solvency of life insurers, there is a rationale for government regulation of long-term insurance, which has been recognised in the UK since a series of insolvencies of life insurers led to the 1869 Insurance Act.[5] The basis of subsequent prudential regulation is the requirements of larger reserves for riskier products so that, even if the life insurer is not risk averse, it may still have to behave as if it is.[6] Indeed, the regulator may also encourage life insurers to price conservatively. For example, in 2007, the chairman of the Financial Services Authority wrote to life insurers recognising that companies would usually make assumptions based on their own mortality experiences, but adding

> "...if this is not possible we would expect firms to consider the different industry views in this area and to err on the side of caution." (FSA Dear CEO letter, April 2007)

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[3] Life insurers must provide detailed accounts to the regulator referred to as the FSA Returns. Where investing in corporate bonds results in a higher yield (a risk premium), life insurers are not allowed to use this to value their liabilities. For example, see the note in *Norwich Union Annuity Limited*, Annual FSA Insurance Returns for the year ended 31st December 2005 (page 53): "In accordance with PRU 4.2.41R, a prudent adjustment, excluding that part of the yield estimated to represent compensation for the risk that the income from the asset might not be maintained, . . . was made to the yield on assets." The return goes on to say that AAA-rated corporate bonds had yields reduced by 0.09 per cent, A-rate by 0.32 per cent and commercial mortgages by 0.41 per cent.

[4] In the U.K., where inflation-adjusted annuities are sold, it is possible to hedge indexed annuities by purchasing government bonds that are indexed to the Retail Price Index. The FSA Returns make explicit that the different types of annuities are backed by different assets. For example, the note in *Norwich Union Annuity Lim*ited, Annual FSA Insurance Returns for the year ended 31st December 2005 (page 50): "Non-linked and index-linked liabilities are backed by different assets and hence have different valuation interest rates."

[5] More recently a leading insurer (Equitable Life) became insolvent in 2000, resulting in the government ultimately agreeing to compensate pensioners in 2010. Plantin and Rochet (2009) analyse the appropriate role and design of prudential regulation of insurance companies.

[6] Text books such as Booth et al (2005) say explicitly that, actuaries have always taken risk into account when pricing annuities.

The prudential regulations have been strengthened by the EU-wide changes to insurance regulation enshrined in *Solvency II,* which will take effect from 2016.[7] Solvency II applies to the insurance industry the risk-sensitive regulatory approach adopted in the Basel reforms for the banking industry. Under the proposal for Solvency II, life insurance companies are required to allow explicitly for uncertainty in their valuations:

> "the technical provision under the Solvency II requirement is the sum of the best estimate and the risk margin, . . ., the best estimate is defined as the probability-weighted average of future cash flows . . . The probability-weighted approach suggests that an insurer has to consider a wide range of possible future events: for example, a 25% reduction in mortality rates may have a small probability of occurrence but a large impact on the cash flows. However, the assumptions chosen to project the best estimated cash flows should be set in a realistic manner, whereas the prudent allowance for data uncertainty and model error should be taken into account in the risk margin calculation." (Telford et al, 2011; paras. 7.2.1 - 7.2.2.3).

In the UK each life insurer must declare the actuarial assumptions used to value its liabilities, by comparing the mortalities (approximately one-year death probabilities) used in its own calculations with the mortalities in the benchmark tables produced by the Institute of Actuaries' Continuous Mortality Investigation (CMI). The CMI collects data from all of the major life insurers: aggregates, anonymises and then analyses the pooled data. So the CMI tables of mortality approximate to the average mortalities across the whole industry. The figures presented in life insurers' FSA returns are then compared to this average and are summarised in Table 1 and illustrate in Figure 1.[8]

[Table 1 and Figure 1 about here]

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[7] http://ec.europa.eu/internal_market/insurance/docs/solvency/131002_draft-directive_en.pdf

[8] The CMI tables include four benchmark life tables for different annuity groups: PCMA00, RMC00, RMV00 and PPMC00. PCMA00 reports the mortalities of members of occupational defined-benefit pension schemes administered by life insurers; RMC00 and RMV00 summarise the mortality evidence of the original DC pensions - retirement annuity contracts for self-employed workers; RMV is for pensioners in receipt of a pension ("vested") and RMC is for both pensioners in receipt of a pension and for those still making contributions ("combined"); and PPMC00 reports mortalities of DC personal pensioners. Using a different benchmark would not affect our conclusions.

Figure 1 shows that for ages above 68, every life insurer assumes lower mortality rates than the benchmark. Some of the variation in assumptions between companies must be due to genuine variations in mortality of the annuitants, but it is obviously impossible that every company has lower mortality than the average, represented by the benchmark. This is *prima facie* evidence that firms are building some allowance for mortality risk into their valuations.

In section 3 we show theoretically how prudential reserving requirements will affect annuity prices. Since there is no disclosure requirement on the value of cohort mortality risk assumed by life insurers, we cannot measure the effect on annuity prices from information in the life insurers' returns. In section 4 we quantify mortality risk through the widely-used model of Lee and Carter (1992) to show that the effect is large enough to affect MW tests in practice.

3. Money's Worth Calculations

In this section we define the money's worth of an annuity product and we furnish the theoretical proof that it will not equal one if the researcher uses a different life table from the life insurer, either due to adverse selection or risk. We then describe the data available on annuity price quotes over the period 1994-2012 and estimate the money's worth for this period.

3.1 The Money's Worth

The conventional measure of the value of an annuity is the money's worth (Warshawsky, 1988; Mitchell et al, 1999), which compares the expected present value of the annuity payments with the price paid for the annuity. Consider the expected present value of a stream of annuity payments, starting with a unit payment and then rising by an escalation factor $g \in \{0, 0.05\}$ per period for an annuity sold to someone age x at time t

(1) $$a_{x,t}(g) \equiv \sum_{i=1}^{\infty}(1+g)^i R_{t,i} s_{x+i|x,t} \qquad s_{x+i|x,t} \equiv \prod_{j=0}^{i-1} p_{t+j,x+j}$$

where $R_{t,i}$ is the discount factor at time t for a pure discount bond of duration i and $p_{t+j,x+j}$ is the one-period survival probability for the annuitant who is age $x+j$ in period $t+j$ (that is the probability of living one more period conditional on being alive at the beginning of the period) and $s_{x+i|x,t}$ is thus the probability of someone aged x at time t living to age $x+i$ or longer. In the absence of administrative and marketing costs, $a_{x,t}(g)$ would be a life insurer's liability from selling a life annuity of unit payments and hence the annuity rate offered by the life insurer would be

(2) $$Annuity\ rate\ (g)_{x,t} = \frac{1}{a(g)_{x,t}^{Life\ insurer}} = \frac{1}{\sum_{i=1}^{\infty}(1+g)^i R_{t,i} s_{x+i|x,t}^{Life\ insurer}}$$

where we have added the super-script "Life insurer" to s to emphasise that this is the survival probability used by the life insurer (calculated at time t, but we leave that implicit for notational simplicity). To calculate the MW, a researcher would use the formula

(3) $$MW\ (g)_{x,t} = Annuity\ rate\ (g)_{x,t} \times a(g)_{x,t}^{Researcher}$$

$$= \frac{\sum_{i=1}^{\infty}(1+g)^i R_{t,i} s_{x+i|x,t}^{Researcher}}{\sum_{i=1}^{\infty}(1+g)^i R_{t,i} s_{x+i|x,t}^{Life\ insurer}}$$

which makes it explicit that the researcher may use different survival probabilities from a life insurer. We assume that the researcher correctly identifies the discount factors used by life insurers and therefore do not distinguish between the discount factors used in pricing the annuity or evaluating the MW.

[Figure 2 about here]

With respect to the difference in survival probabilities, we illustrate two possible cases in Figure 2. In the first case (Panel A), we assume that there are two types of individual, high and low risk, who know their type and know that they have different survival probability curves. In an adverse-selection separating equilibrium, high-risk types choose escalating annuities ($g = 0.05$) and low-risk types choose level annuities ($g = 0$). Since the type is revealed by annuity choice, the life insurer is able to use the correct survival probabilities in pricing the annuity.

However, the researcher is faced with using data provided by the CMI, which only publishes one set of life tables, not distinguishing annuitants with different types of annuity. This pooled life table will have a survival probability denoted by the heavy black line in Panel A which lies between the low and high-risk types' curves. Hence the researcher will systematically under-estimate the survival probability when calculating the MW of annuities purchased by high-risk individuals and over-estimate the survival probability of annuities purchased by low-risk individuals. The estimated MW for level and escalating annuities would be

$$MW\ (g=0, low\ risk)_{x,t} = \frac{\sum_{i=1}^{\infty} R_{t,i} s_{x+i|x,t}^{Average}}{\sum_{i=1}^{\infty} R_{t,i} s_{x+i|x,t}^{Low\ risk}}$$

(4)

$$MW\ (g=0.05, high\ risk)_{x,t} = \frac{\sum_{i=1}^{\infty} 1.05^i R_{t,i} s_{x+i|x,t}^{Average}}{\sum_{i=1}^{\infty} 1.05^i R_{t,i} s_{x+i|x,t}^{High\ risk}}$$

We now demonstrate that the MW of the escalating annuity ($g = 0.05$) will be lower than that of the level annuity ($g = 0$).

Proposition 1: Where a life insurer sells two annuity types in a separating equilibrium, then $MW(g = 0.05, high - risk)_{x,t} < MW\ (g = 0, low - risk)_{x,t}$

Proof: We assume that the survival curves for high- and low-risk never cross for which a sufficient condition is that the mortality of a high-risk individual is always lower than the mortality of a low-risk individual of the same age in the same year. In which case the survival probabilities can be ordered as follows

$$\forall i\text{:}\ s_{x+i|x,t}^{Low-risk} < s_{x+i|x,t}^{Average} < s_{x+i|x,t}^{High-risk}$$

It then follows that

$$\forall i\text{:}\ R_{t,i} s_{x+i|x,t}^{Low-risk} < R_{t,i} s_{x+i|x,t}^{Average}, \qquad and \quad 1.05^i R_{t,i} s_{x+i|x,t}^{Average} < 1.05^i R_{t,i} s_{x+i|x,t}^{High-risk}$$

Where we assume that the same interest rates are used by both the life insurer and the researcher, but where the life insurer is pricing annuities using either the high- or low-risk survival probabilities as appropriate, and the researcher uses the average survival probability. Summing over all future years:

$$\sum_{i=1}^{\infty} R_{t,i}\, s_{x+i|x,t}^{Low-risk} < \sum_{i=1}^{\infty} R_{t,i}\, s_{x+i|x,t}^{Average},$$

$$and \sum_{i=1}^{\infty} 1.05^i R_{t,i}\, s_{x+i|x,t}^{Average} < \sum_{i=1}^{\infty} 1.05^i R_{t,i}\, s_{x+i|x,t}^{High-risk}$$

$$\Rightarrow 1 < \frac{\sum_{i=1}^{\infty} R_{t,i}\, s_{x+i|x,t}^{Average}}{\sum_{i=1}^{\infty} R_{t,i}\, s_{x+i|x,t}^{Low-risk}}, 1 < \frac{\sum_{i=1}^{\infty} 1.05^i R_{t,i}\, s_{x+i|x,t}^{High-risk}}{\sum_{i=1}^{\infty} 1.05^i R_{t,i}\, s_{x+i|x,t}^{Average}}$$

$$\Rightarrow 1 < \frac{\sum_{i=1}^{\infty} R_{t,i}\, s_{x+i|x,t}^{Average}}{\sum_{i=1}^{\infty} R_{t,i}\, s_{x+i|x,t}^{Low-risk}} \times \frac{\sum_{i=1}^{\infty} 1.05^{i} R_{t,i}\, s_{x+i|x,t}^{High-risk}}{\sum_{i=1}^{\infty} 1.05^{i} R_{t,i}\, s_{x+i|x,t}^{Average}}$$

$$\Rightarrow 1 < \frac{MW\ (g=0, low-risk)_{x,t}}{MW(g=0.05, high-\ risk)_{x,t}} \qquad \text{QED}$$

In the second case, illustrated in Panel B of Figure 2, we assume there is no adverse selection and no separating equilibrium but the life insurer is uncertain of future values of the relevant survival probabilities. In line with the Solvency II framework, we use the Value-at-Risk (VaR) as a guide to suitable reserving, by which we mean that insurers price off the tail of probability distribution of future mortality such that there is a 95 per cent chance of having sufficient assets to meet the actual risky liabilities. We plot the forecast survival probabilities with a central projection and upper and lower confidence intervals. In the conventional MW calculations, researchers implicitly use the central projection as the price of the annuity contract, but a risk-averse life insurer would price using the upper confidence interval and hence the MW would be

$$MW\ (g=0)_{x,t} = \frac{\sum_{i=1}^{\infty} R_{t,i} s_{x+i|x,t}^{Average}}{\sum_{i=1}^{\infty} R_{t,i} s_{x+i|x,t}^{Upper\ C.I.}}$$

(5)

$$MW\ (g=0.05)_{x,t} = \frac{\sum_{i=1}^{\infty} 1.05^{i} R_{t,i} s_{x+i|x,t}^{Average}}{\sum_{i=1}^{\infty} 1.05^{i} R_{t,i} s_{x+i|x,t}^{Upper\ C.I.}}$$

As with the adverse selection case, part of the problem is that the researcher will use the "wrong" survival probabilities; conversely in this case the life insurer uses the same survival probability curve to value both annuity types. The relationship between these two money's worths depends on the uncertainty of the forecast of future survival probabilities, and the extent to which life insurers' price annuities to take account of mortality uncertainty. Define the range between the average and the upper confidence interval of the survival distribution function by the "concordance ratio", φ_i, for each future year i, where

$$\varphi_i = \frac{s_{x+i|x,t}^{Average}}{s_{x+i|x,t}^{Upper\ C.I.}}$$

and where we omit x and t subscripts for notational simplicity. If there were no uncertainty about future survival probabilities, and if life insurers priced at the central projection, then $\varphi_i = 1$; but when there is uncertainty about future survival probabilities, then $0 < \varphi_i < 1$, and lower values of φ_i are associated with more uncertainty. [9] We shall be interested in the scenario: $\varphi_{i+1} < \varphi_i$ i.e. where uncertainty increases with the time horizon, and hence the concordance ratio falls as projections are made at higher ages. There are three reasons for the uncertainty of survival probabilities to increase over time. First, as is conventionally the case, forecast error increases with time horizon. Second, actuarial models forecast mortality, but the variable of interest is survival probability: survival probabilities further into the future compound a larger number of uncertain death probabilities. [10] Third, regardless of the model used, survival probabilities immediately after the sale of an annuity will be very close to one: in our data the crude death rate for 65-year-olds over the period 1983-2000 is 0.018. The one-year survival probability is then 0.982, and the concordance ratio thus cannot be less than 0.982 because the upper confidence interval probability cannot exceed one. As survival probabilities fall this constraint is relaxed, and so the concordance ratio can decrease. Using the definition of the concordance ratio into (5), then

$$MW(g) = \frac{\sum_{i=1}^{\infty}(1+g)^i R_i s_i^{Upper\ C.I.}\left(s_i^{Average}/s_i^{Upper\ C.I.}\right)}{\sum_{i=1}^{\infty}(1+g)^i R_i s_i^{Upper\ C.I.}} = \sum_{i=1}^{\infty} w_i(g)\,\varphi_i$$

where

$$w_i(g) \equiv \frac{(1+g)^i R_i s_i^{Upper\ C.I.}}{\sum_{j=1}^{\infty}(1+g)^j R_j s_j^{Upper\ C.I.}}, \quad \sum_{i=1}^{\infty} w_i(g) = 1$$

i.e. MW is a weighted average of our measure of the concordance ratios.

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[9] The concordance ratio is inversely related to the uncertainty in the survival probabilities. We plot a sample concordance ratio for our estimated model in Appendix A.2, which illustrates that this ratio is falling (uncertainty increasing) with the time horizon.

[10] Taking two specific cases from equation (1): $s_{x+1} = (1 - q_1)$, which depends on one random variable but $s_{x+2} = (1 - q_1)(1 - q_2)$ depends on two random variables.

Proposition 2: Where there is no selection and life insurers prudently price annuities from the upper confidence interval, then $MW(g = 0.05) < MW(g = 0)$ if $\varphi_{i+1} < \varphi_i$.

Proof: From the definition of $w_i\ (g)$, an increase in g reduces $w_i(g)$ for smaller values of i and increases $w_i(g)$ for larger values of i. Since $\varphi_{i+1} < \varphi_i$, MW is a weighted average of the concordance ratios. An increase in g adjusts the weights so that more weight is placed on the values of φ_i at longer horizons where φ_i is smaller (i.e. uncertainty is greater) and hence MW falls. QED

Ceteris paribus, an increase in g corresponds to an increase in the expected duration of the life annuity: it is more "back-loaded". These two propositions show that a researcher using publicly available data would find the money's worth for a back-loaded annuity to be less than that of a level annuity: this difference may be due either to an adverse-selection separating equilibrium or to prudential pricing. Our proof that prudential pricing (Proposition 2) results in a lower money's worth for escalating annuities depends on the fact that uncertainty increases with time horizon, and back-loaded annuities have a great percentage of the present value at longer time horizons. The proof for selection (Proposition 1) depends on the fact that higher-risk individuals should choose back-loaded annuities. The two propositions share the intuition that the calculated money's worths will differ: (i) because the researcher uses a different set of survival probabilities from the life insurer; and (ii) because real and escalating annuities have longer duration than level annuities.

3.2 Description of the data

Data on UK annuity rates for males at various ages are taken from MoneyFacts over the period August 1994 to April 2012, and an average monthly value is computed which corresponds to the annuity rate in equation (2). These are compulsory-purchase annuities which are bought in the decumulation phase of a defined contribution pension scheme. [11] The discount factor $R_{t,j}$ (which we assume to be risk free) may be inferred from the yield curve on government bonds at the time of the annuity sale. This is likely to be a good approximation to life insurers' rates we have noted above that life insurers predominantly back their liabilities with government bonds and have to adjust rates of return on other assets for risk so this must be a good approximation to the rates they use.[12] Secondly, life insurers approximately match their annuity liabilities with government bonds.[13]

In Figure 3 we illustrate the annuity rate series for a 65-year old male over time compared with government bond data, and summary statistics of these data for nominal and real variables are presented in Tables 2 and 3. It can be seen that nominal annuities approximately track the nominal bond yield and analogously for real annuities: annuity rates are highly correlated with long-term bond yields, and the average difference in these two series over the sample period was 2.86%. We also compare the two sub-periods up to the financial crisis (Northern Rock bank run in August 2007) and since the onset of the crisis. Following the crisis, both short-term (base rate) and long-term government bond yields have fallen, and this has been reflected in a fall in annuity rates. Level annuities pay a constant annuity payment in nominal terms throughout the lifetime of the annuitant; real annuities have payments that rise in line with the UK's Retail Price Index, and escalating annuities incorporate an escalation factor of five per cent per annum.

[Tables 2 and 3, and Figure 3 about here]

The remaining data that we need to estimate the money's worth are the mortality projections. We used a series of life tables for annuitants published by the CMI, and

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[11] For example, in the UK in July 2009, the Prudential would sell an annuity for £10,000 to a 65-year old man which would pay a monthly income of £61, or £732 annually for life: the annuity rate would be $A_{2009,65} = 732/10{,}000 = 7.32\%$.

[12] Details of the notional yields, credit ratings and corresponding adjustments are reported in the FSA returns; see also footnote 3. Price risk is relatively unimportant since bonds are typically held to maturity.

[13] CGFS (2011) provides a review of international insurance regulation and notes that this matching can be *duration matching* which only partially matches liability and asset cash flows and *cash-flow matching* which perfectly matches the flows. The footnotes of various FSA returns note that perfect matching is impossible and that there is a small residual risk.

started using each new table from a year before the publication date, on the argument that the broad outline of these data may have been known to life insurers before actual publication (and life insurers would also have been able to analyse the mortality experience of their own annuitants). The PML80 ("Purchased Male Life") table was published in 1992 ("80" refers to the base year). Although it projected gradual increases in life expectancy, by the late 1990s it had become clear that the downward trend in mortality of pensioners was much stronger and the PML92 tables (published 1999) revised life expectancy up by almost two years. Further analysis of the reduction in mortality both for pensioners and people of below pension age (for which pension data were unavailable: life insurance data were used instead), suggested a "cohort" effect, i.e. a discrete downward jump in mortality for people born after about 1930. This led to a set of "interim adjustments" published in 2002: the most widely used "medium cohort" adjustment is illustrated here. In 2005 information on the most recent annuitant mortality was published (the "00" table), which did not have an accompanying projection for changes into the future. Accordingly at that time many life insurers used the "00" table as a base and then used the "medium cohort" projection from 2000 (or some other year) onwards.

3.3 Estimates of the (Conventional) Money's Worth

Figure 4 illustrates the MW of the monthly annuity rate data for men in the UK compulsory purchase market for three different ages (60, 65, 70,). We calculate the MW using the mortality projections from the relevant CMI tables for each period, with a short overlap. It can be seen that each new actuarial table results in a discrete increase in the MW due to longer projected life expectancy, but the medium cohort projection and the PNML00 projection match almost exactly. Within the sample period for a particular mortality table there is an apparent decline in MW for males of all ages, with a spike around 2008 reflecting low bond yields and downward shift in the nominal term structure, and a delayed reaction in terms of reduced annuity rates. The range of money's worths across the three ages fell considerably over time.

[Figure 4 about here]

Although Figure 2 showed a decline in annuity rates of about 2.5 per cent between 1994 and 2000, Figure 4 shows that this does not correspond to as large a change in

MW: this fall is mainly explained by falls in interest rates and increases in life expectancy. Table 4 provides formal tests of the differences in money's worths by age, guarantee period, and annuity product type, over the four sub-periods of our data corresponding to the relevant actuarial life table. In Panel A of Table 4 we compute the average MW by age, and examine whether there are significant differences between the MW of annuities at different ages. We test for the equality of means of these series, using a "matched pair" analysis to deal with trends in the series. We calculate the t-statistic for the mean value of these differences, using Newey-West standard errors, with the relevant adjustment for the autocorrelation structure. The reversal of MW by age over the period 2001-2004 for 70-year old males (t-stat on difference with 65-year old males is -1.94) is inconsistent with the suggestion of F&P (2002, p. 41) that lower MW at higher ages is evidence for asymmetric information. [14]

[Table 4 and Figures 5 and 6 about here]

Figure 5 and Panel B of Table 4 shows that there is little difference in MW for annuities with different guarantee periods. Figure 6 and Panel C of Table 4 reports the MW for level, real and escalating annuities: we are able to confirm that back-loaded annuities have significantly lower money's worths than level annuities for each of the sub-samples. For example, for the most recent table 2004-2012 (the "00" table), has MW for real and escalating annuities as 0.768 and 0.802 respectively, and the MW for level annuities as 0.859. Note that the MW of real annuities display a negative spike in 2008, which is due to the perverse movement in real bond yields at that time, as is clear from Figure 3. Comparing the beginning of the period to the end (the two periods when we are relatively confident about the appropriate mortality table to use), there is some slight evidence that MW has fallen and that the gap between the nominal and real money's worth has risen. The results for the relative MW of real and escalating are more mixed: the gap between them is often small and sometimes the MW for real annuities is slightly higher than for escalating,

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[14] Cannon, Stevens and Tonks (2013) analyse the Dutch annuity market and also find an inverse pattern of money's worths by age for the period 2001-2010.

rather than lower. Overall, our analysis for the money's worth over the whole period largely confirms that of F&P (2002).[15] The caveats are that the differences in MW by age or guarantee have disappeared by the end of the period.

4. The Stochastic Money's Worth

In the section 3.1, we showed the pattern of observed money's worths arising from life insurers reserving against cohort mortality risk. Since life insurers do not report how they reserve for this risk, we quantify the effect by estimating the uncertainty in forecasting the probability of living $p_{t+j,\, x+j}$ (equivalently the probability of dying) and determining the amount of reserves needed when calculating the annuity price using the Value-at-Risk (VaR) approach discussed in section 3.[16]

The estimation of death probabilities is a staple of actuarial textbooks (Bowers *et al,* 1997; Pitacco et al, 2009), but forecasting these variables is more problematic and usually relies on extrapolating the past trend, because models based on the causes of death are insufficiently precise to be used for prediction purposes. The very long-term nature of these forecasts, results in estimates that are subject to uncertainty from a variety of sources.

First, there are issues with the timeliness and quality of the historical data. The estimates are based on data available only up to time t (or possibly earlier if there are lags in data collection): in many countries sufficiently detailed data for p are simply unavailable and the U.K. is unusual in having reliable data for pensioners over a long time period. Since 1924, U.K. life offices have provided their firm-level data on survival experiences to a central committee of actuaries to create a large enough data set to enable reliable statistical analysis and long-term projections. A second problem is that observed death rates are only estimates of the underlying death probabilities due to sampling error; this may be particularly acute when only small samples are available, which is often the case for the highest ages. A third

[15] A robustness check on the differences in the log-money's worths is provided in Appendix A.1.

[16] More formally, mortality μ is the continuous-time analogue of the one-year death probability $q \equiv 1 - p = \int d\mu$. In this paper we work entirely with one-year death probabilities and ignore the issue of when deaths occur within year: the quantitative effect of this is very small.

issue is that there may be structural changes in the data generating process associated with healthcare improvements over time, including: universal changes in health technology affecting all cohorts; the health of annuitants changing relative to that of the general population; changes in health due to lifestyle changes; or changes in the health of pensioners due to changes in pension coverage. In addition, such changes may have led to selection effects in the types of people enrolling into a pension scheme in the first place.

Our estimated mortality models use the UK's life office pensioner mortality data, which is the largest and most commonly used data set for UK private pensions, for the years 1983-2000:[17] the typical exposed to risk for a given age in a given year is in the range 5,000-10,000, although there are fewer for very high ages. The total exposed-to-risk in 1983 is 356,552 and in 2000 it is 289,019.

The cohort mortality model we use for our application is Lee and Carter (1992), which has been widely accepted as a starting point for mortality analysis.[18] Cairns et al (2011) consider the forecasting performance of a range of mortality models, and by focusing on the uncertainty within the Lee-Carter model we are probably under-estimating the effect of model uncertainty on the money's worth for two reasons. First, we make no allowance for different life insurers using different models which may add to the model uncertainty, and second within the class of mortality models Cairns et al (2011) note that the Lee-Carter model produces forecasts that are 'too

17 Although detailed data on pensioner mortality were collected in the United Kingdom from 1948 the data prior to 1983 have been lost (CMI, 2002). In this data set no 60-year old male died in 1998, so the log mortality was not defined: we replaced the zero value by 0.5 (which corresponded to the lowest mortality rate observed elsewhere in the data set). A variety of alternative assumptions resulted in almost identical conclusions.

18 Most mortality models' starting point is Gompertz's Law: that the logarithm of death rates tends to increase linearly with age. In addition, the log of death rates decreases linearly over time. Caveats to these statements are: these relationships are only approximately linear; that falls over time in mortality may be age dependent; and there are occasional structural breaks. There are also issues as to whether one should look at the logarithm of the death rate or an inverse logistic function, and whether the decline is a stochastic or deterministic trend (Cairns et al, 2009). In Table 6 below, as a robustness check we consider an alternative to the Lee-Carter approach: the Cairns-Blake-Dowd (2006) model, which uses the approximately linear relationship between log-mortality and age as a restriction in the estimation strategy. Details of estimating the Cairns-Blake-Dowd model and the expected annuity values from different mortality models and different sub-samples of the data, are provided in Appendices A.3 and A.4 respectively.

precise' compared with the historical volatility of mortality; so our estimates of uncertainty are probably conservative. Lee and Carter (1992) model the one-year death probabilities as

(6) $$\ln(1 - p_{xt}) = \ln q_{xt} = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt} \quad \varepsilon_{xt} \sim N(0, \sigma^2)$$

which can be estimated by Least Squares (LS) from a singular-value decomposition method. Pitacco et al (2008) and Girosi and King (2008) suggest that LS was the most widely used estimator and we think it unlikely that life insurers would have used Maximum Likelihood (ML) during the period for which we have annuity rate data, but we report ML results for comparison. Our baseline results are estimated for ages 61-100 for the period 1983-2000 but for robustness we also estimate models for ages 60-100 and 65-100. An explanation of the technical issues implementing the Lee-Carter model are discussed in Appendix A.2. Regardless of the estimation procedure, forecasting is based upon a stochastic trend

(7) $$\Delta\kappa_t = \lambda + \psi_t \qquad \psi_t \sim iid(0, \sigma_\psi^2).$$

where the parameters λ and σ_ψ^2 are estimated in a second-stage regression. As a robustness check we also consider a model where parameter κ_t follows a deterministic trend (Girosi and King, 2008).

[Figure 7 about here]

The results of our baseline estimates are illustrated in Figure 7. Consistent with Gompertz's law the alphas and betas are approximately linear in age, and the kappa follows something close to a stochastic trend. The fact that beta depends upon age shows that the trend in log-mortality is age dependent.

Using the estimated alphas and betas and with projected kappas, we can project survival probabilities into the future using numerical methods: we conduct Monte Carlo experiments with 10,000 replications to calculate the probability distribution of the relevant stochastic variables (details in Appendix A.2). Figure 8 shows the survival fan chart for a male aged 65 at the end of the period of our data in 2001. Such fan charts have been discussed in Blake, Dowd and Cairns (2008): there is relatively little uncertainty about the survival probability for the first few years, when the probability of dying is small and there is little scope for uncertainty.

However, by age 75 there is considerable uncertainty. Note that an annuity which was more back-loaded (had longer duration) would have a higher proportion of its present value paid in the period of greater uncertainty and thus would be a riskier liability for a life insurer.

[Figure 8 about here.]

Underlying the calculations which generate the survival probabilities in Figure 8 we have 10,000 paths for the survival probabilities and we evaluate the corresponding annuity value of each of these for different interest rates to get estimates of the density function of the value of an annuity (i.e. the value of a in equation (1)).

[Figure 9 about here.]

These density functions are illustrated in Figure 9 for different interest rates, assuming that the yield curve is horizontal. For example, the probability density function when the interest rate is zero has a range between £17.68 and £21.36: if the insurance company priced the annuity on the expected value then the present value of £1 life annuity would be £19.65 and the implied annuity rate would be 5.11 per cent. If the insurance company sold a large number of annuity policies at this price, it would break even in expectation but, given the distribution is approximately symmetric, about half of the time it would make a loss. If the life insurer wished to ensure that it made a profit 90 per cent of the time, it would set the price at the 90th percentile of this distribution which would have an expected present value of £20.20 and the annuity rate would have to be 4.95 per cent, resulting in a MW of 0.97. Figure 9 also shows that, as the interest rate rises and the duration of the annuity falls, both the expected value of an annuity and the standard deviation fall. The riskiness of the distribution falls as interest rates rise, because with higher interest rates, future uncertainty is discounted more heavily.

Table 5 shows the consequences for the money's worth if a life insurer prices annuities from the relevant centile of the distribution of annuity values but the researcher uses the expected annuity value. In this table we compute the money's worth of a £1 annuity, where the expected value of the annuity payments is computed as the discounted sum of annuity payments multiplied by survival probabilities, but where the annuity is priced at either the 50th , 90th, or 95th centile of the distributions given in Figure 9. When priced from the median, MW is approximately one, because the median price and the expectation of the annuity payments are virtually the same. When the life insurer prices from the 90th centile, MW is less than one and the discrepancy is larger the lower the interest rate (because the duration of the annuity rises and is where there is greater uncertainty). At more conservative pricing (95th centile), the money's worth are even lower.

[Table 5 about here]

Panel A of the table shows the effect of changing interest rates and degree of VaR pricing for level annuities on the money's worth of level annuities. Panel B computes MW for escalating annuities, which is one type of back-loaded annuity, and Panel C reports the difference in the MW between level and escalating annuities. We can see from Panel C that at each interest rate, or at each centile of the distribution, the MW of the escalating annuity is lower; and the difference is just under five per cent when annuities are priced at the 95th centile, and interest rates are around five per cent, which is the approximate average value of the 10 year government bond yield over the period 1994-2012. This difference goes some way to explaining the difference in the actual money's worths of level and back-loaded annuities in Panels A and C of Table 2 illustrated in Figure 6.

[Table 6 about here]

In Table 6 we illustrate the robustness of our results to alternative estimation methods and mortality models. The numbers in Table 6 again show the differences between the money's worth for a level and escalating annuities, and where we are assuming that the life insurer prices annuities off the 90th centile. The columns reports the results based on: different sub-samples of the data (ages 60-100, 61-100 and 65-100); different estimation methods (Least Squares or Maximum Likelihood); projections based on either a stochastic (S) or deterministic trend (D); and different mortality models (Lee-Carter and Cairns-Blake-Dowd). The third column in Table 6 repeats the penultimate column of Table 5 Panel C for ease of comparison. In all cases it can be seen that the differences in MW between level and escalating annuities are positive, meaning that irrespective of the mortality model, the data sample or the assumptions about the trend in life expectancies, the money's worth of level annuities is higher than that for escalating annuities when life insurers price at the 90th centile of the annuity value distribution but the researcher uses the expected annuity value.

Although the CBD results in the final three columns of Table 6 suggest a smaller effect on the money's worth than for the Lee-Carter model, comparing these two estimates depends partly on which model is considered the better predictor of annuity values. In a comparison of six mortality models, Dowd et al (2010) provide results suggesting that the CBD model is slightly better at predicting future mortalities but that the Lee-Carter model is better at predicting annuity values.

In Figure 10 we illustrate our final calculations making use of the actual interest rates that were used in the money’s worth calculations in Figures 4-6. The diagram shows time series of money’s worths for level, real and escalating annuities based on an annuity provider pricing off the 90th centile of the annuity distribution. Notice, however, that we are using a constant set of mortality projections for the whole period, so our results are not directly comparable with the earlier graphs. Instead, Figure 10 isolates the effect that actual interest rate changes would have had on MW calculations had annuities been priced on the 90th centile. Figure 10 reinforces our calculations in Table 5: a significant part of the difference between nominal and back-loaded money’s worths is in part due to cohort risk.

[Figure 10 about here]

Comparing the money’s worth of the three product types, the figure shows that while MW for real annuities is less than that for nominal, it is greater than that for escalating annuities, inconsistent with our empirical results in Section 3.2. This inconsistency with the data is exactly that same as that noticed by F&P (2002, pp.45-46): an adverse selection separating equilibrium would also incorrectly predict that the money’s worth for real annuities would be between that of nominal and escalating annuities, given that during the sample period the inflation rate has averaged less than the 5%. While our model is unable to fit the data in this respect, the inconsistency emphasises the difficulty in identifying the two models of annuity pricing: both give the same wrong result since both utilise the feature that real and escalating annuities have longer durations than level annuities.

One additional explanation for the lower money's worth for real annuities is that they have additional idiosyncratic risk: the number of real annuities sold is too small to achieve portfolio diversification and additional calculations suggests that this might reduce the money's worth by an additional one percent. Details are provided in the Appendix A.5. Further, there may be higher costs of managing a portfolio of real bonds and some evidence for this is provided in Debt Management Office (2013).

5. Summary and Conclusions

In this article we have provided estimates of the money's worth calculations for the UK compulsory purchase market, and have shown that the finding in F&P (2002) established from a cross-section of annuity prices in 1998, that back-loaded annuities have a lower money's worth than front-loaded annuities is true over the whole period 1994-2012. F&P explain this as an adverse-selection separating equilibrium achieved by longer-lived individuals purchasing back-loaded annuities.

We have shown that an alternative model yields the same qualitative conclusions. Our model relies upon the fact that life insurers need to reserve against the uncertain evolution of cohort mortality, both for prudential reasons and because they are required to do so by government regulation. Because back-loaded annuities have a higher proportion of pay-outs in the more distant future, they are inherently riskier products and require greater reserves.

Because our model yields the same conclusions as the F&P (2002) model it is impossible to identify the magnitude of the two effects from the data alone. To address this problem we have quantified the importance of cohort mortality risk using the Lee-Carter model. Our results suggest that a substantial proportion of observed differences in money's worths for different annuity products may be due to the relative risk. Combined with other costs of annuity supply, which are conventionally ignored in money's worth calculations, this suggests a much smaller rôle for adverse selection.

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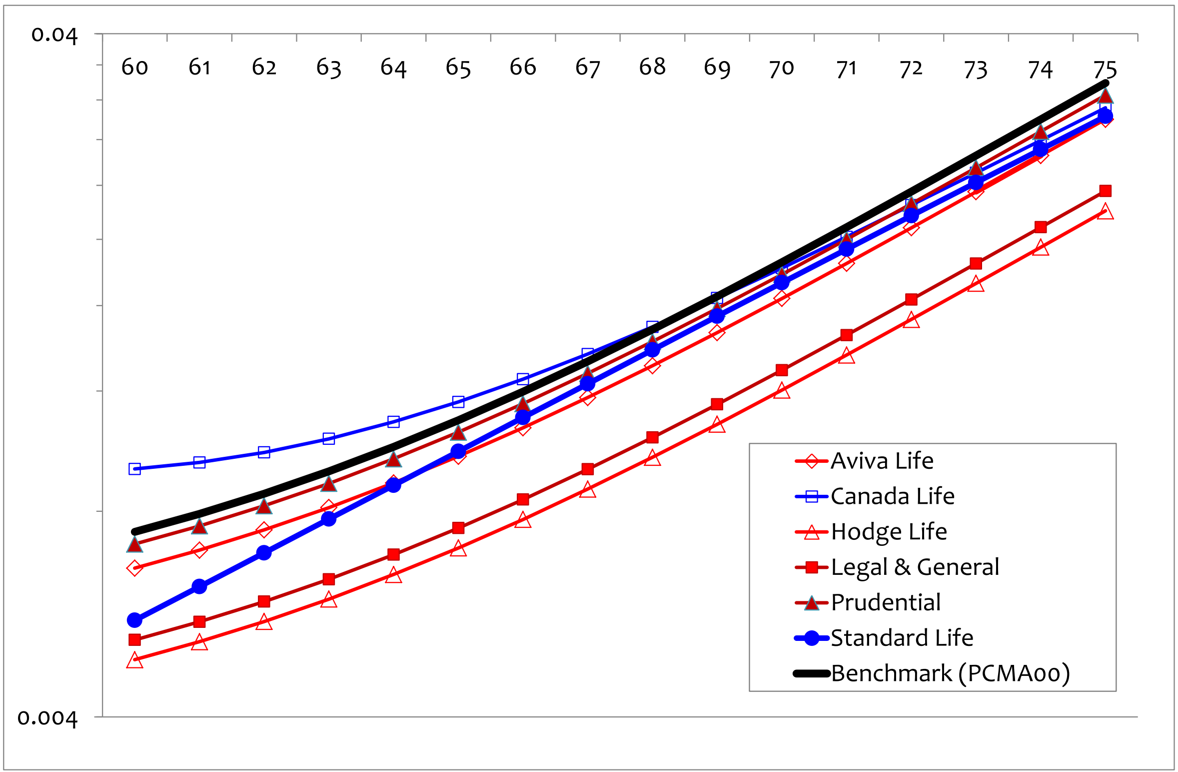
Figures and graphs

Figure 1 Mortality assumptions of life insurers

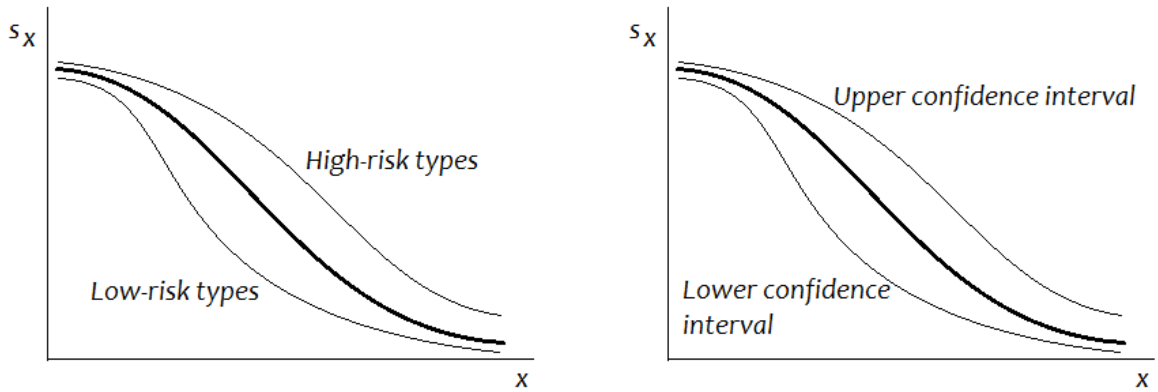


The figure compares the CMI benchmark projected future mortality with the company specific mortality assumptions provided in the FSA returns. For explanation of mortality tables used see footnote to Table 1.

Figure 2: Two models of survival probabilities.

Panel A

Panel B



The figure shows survival probabilities as a function of age. Panel A shows the survival probabilities for two risk types and the average of these survival probabilities. Panel B shows the average survival probability and the upper and lower confidence intervals.

Figure 3: UK Annuity Rates (Male, Compulsory Purchase) and Bond Yields

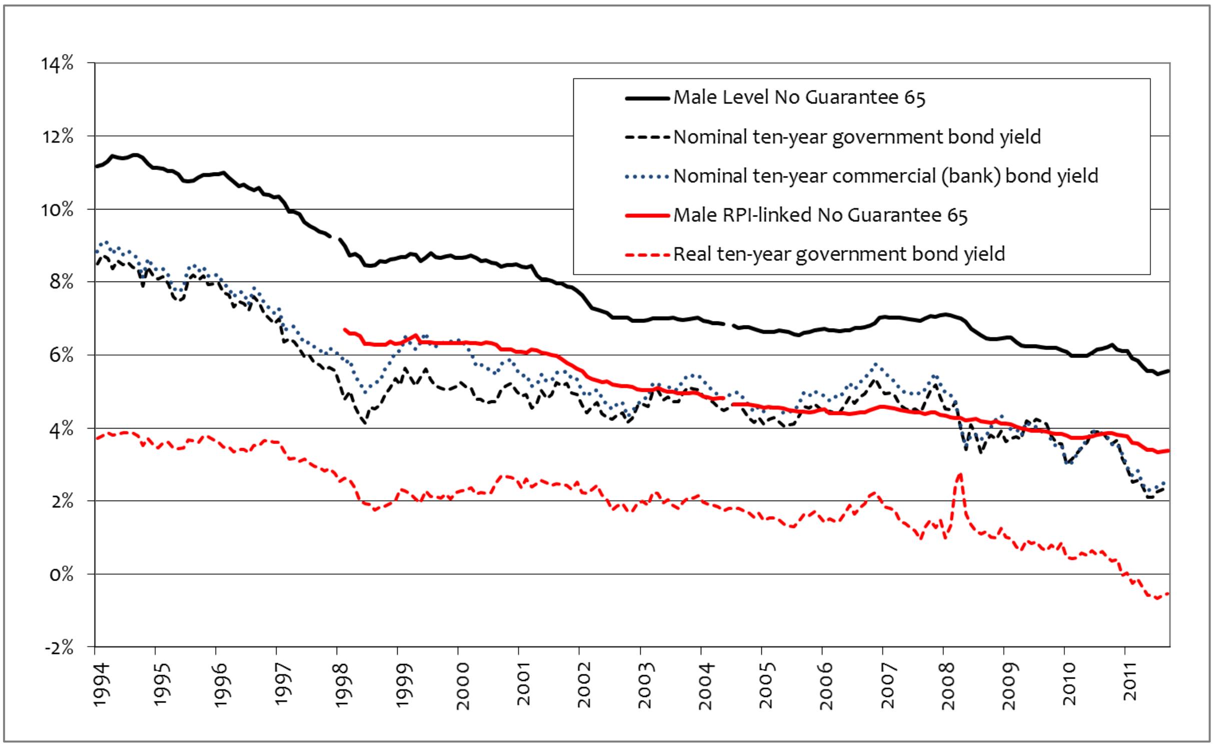


Figure shows monthly time series for 1994-2012 of average annuity rates (across providers) for 65-year old male for level and index-linked annuities (1998-2012); yields on nominal ten-year government and commercial bonds; and real yields on index-linked ten-year government bonds.

Figure 4: Money's worth calculations, level annuities for different ages

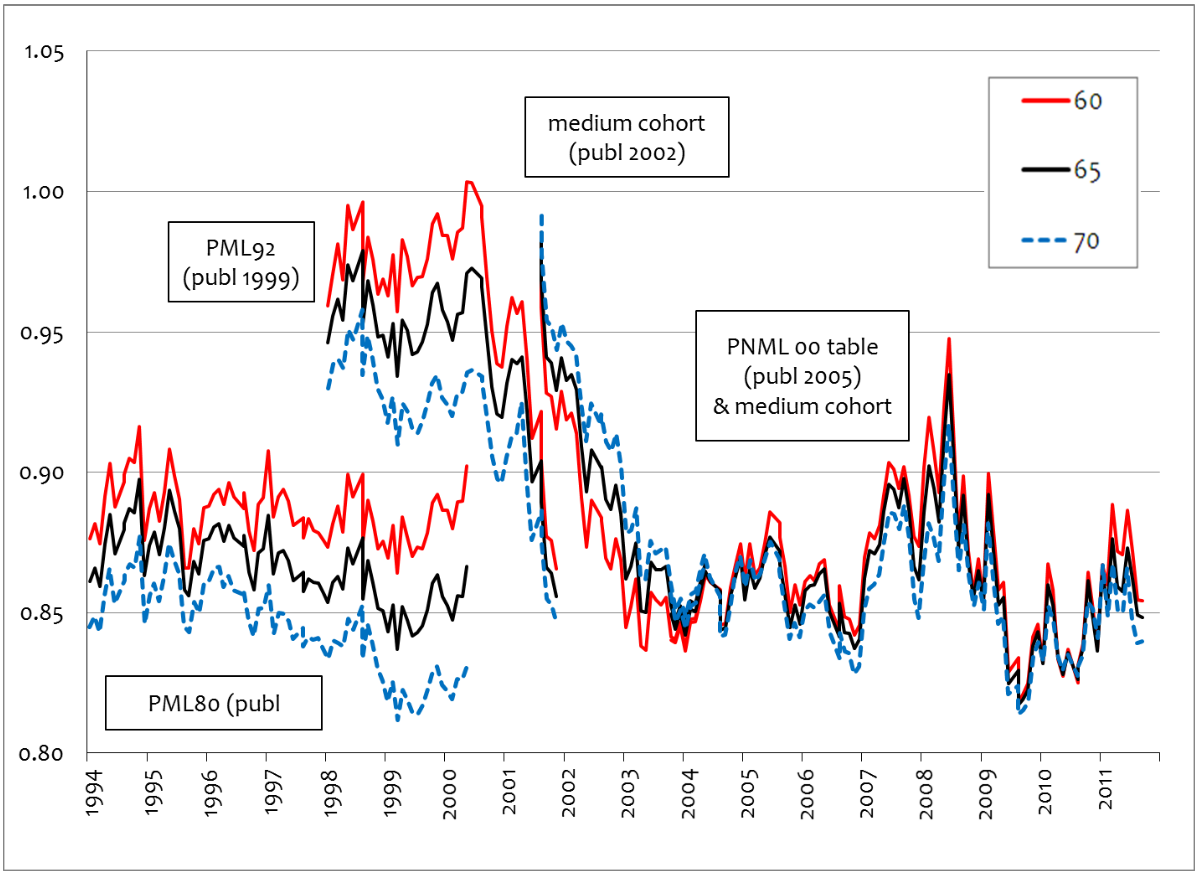


Figure shows MW of level annuities for males aged 60, 65, 70 over four sub-periods corresponding to relevant mortality tables (PML80 refers to data from 1994-2001; PML92 refers to data from 1999-2002; medium cohort refers to data from 2002-2005; and PNML00 refers to data from 2005 to 2012).

Figure 5: Money's worth calculations, different guarantee periods, male, 65

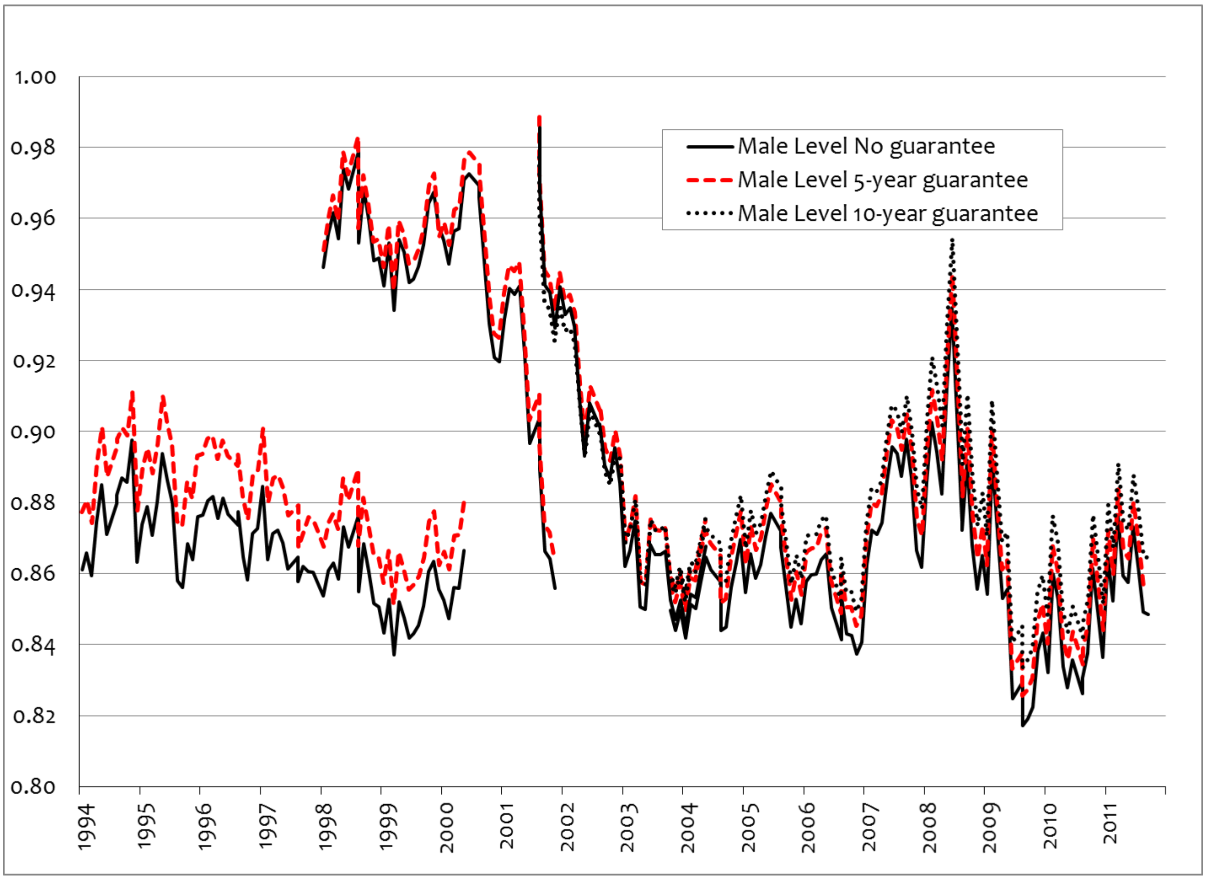


Figure shows MW of level annuities for males aged 65 by guarantee (none, 5-year, and 10-year guarantee), over four sub-periods corresponding to relevant mortality tables; see footnote to Figure 4.

Figure 6: Money's worth calculations, different types of annuity, male 65

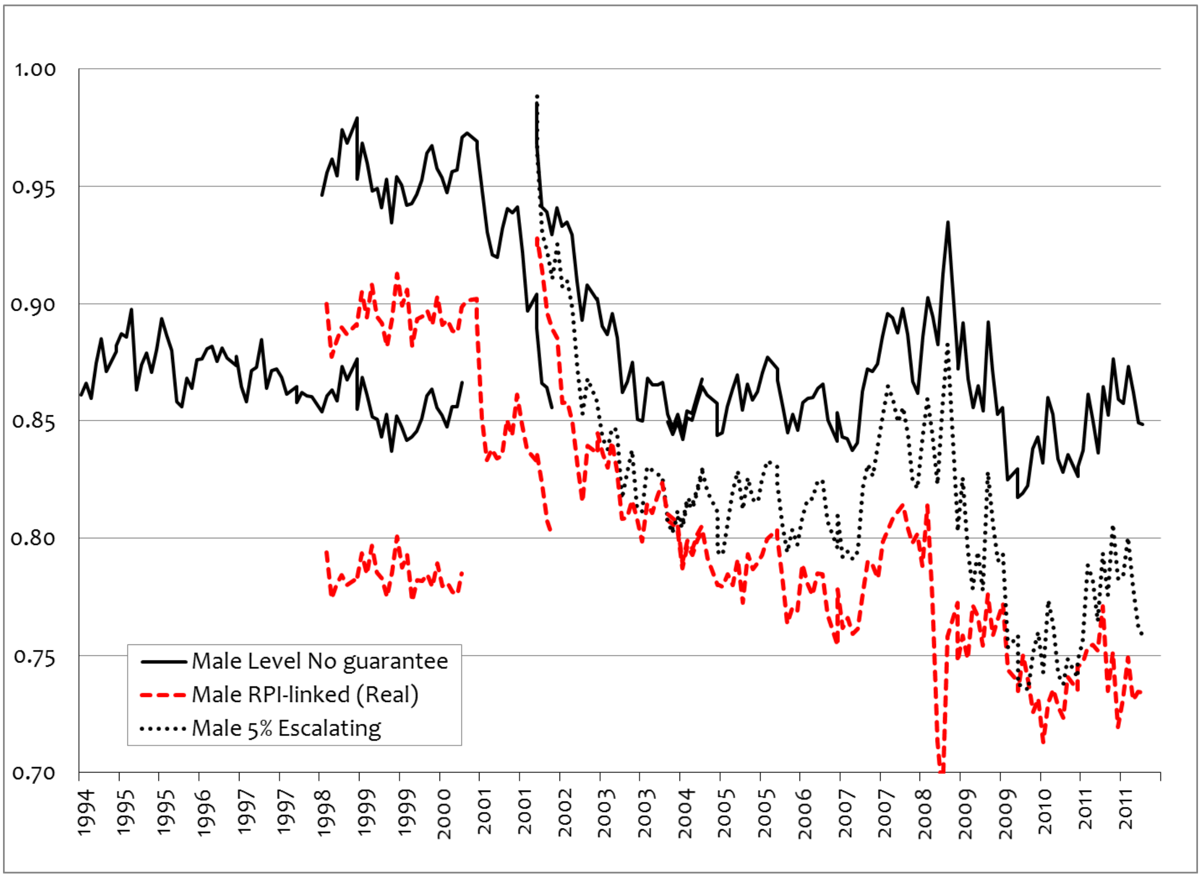
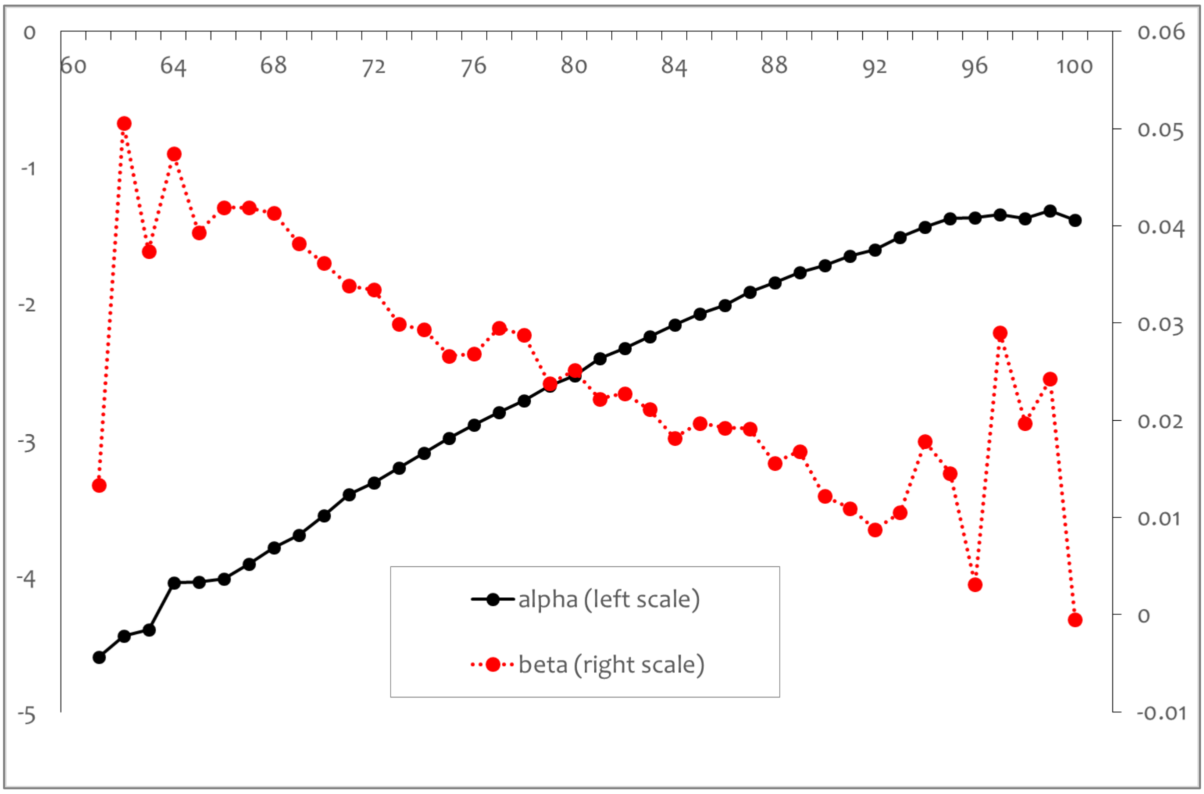
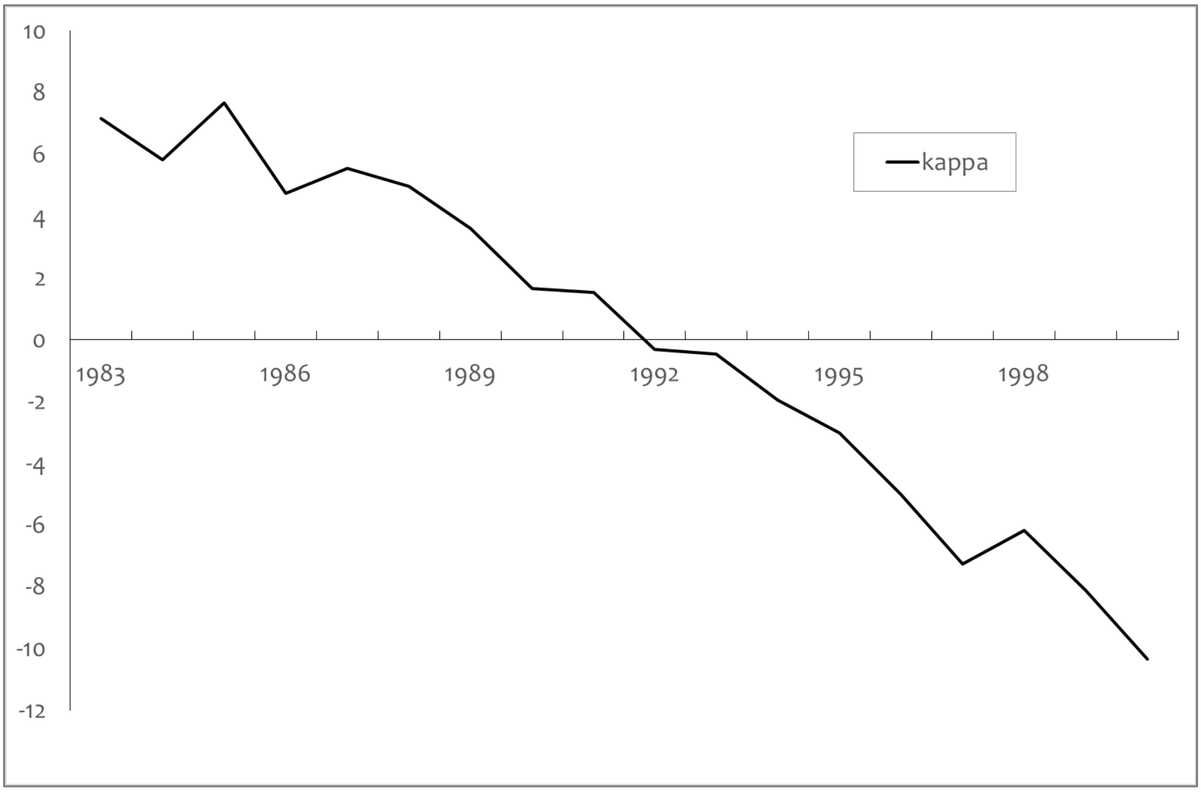


Figure shows MW of level, index-linked (real) and escalating annuities for males aged 65, over four sub-periods corresponding to relevant mortality tables; see footnote to Figure 4.

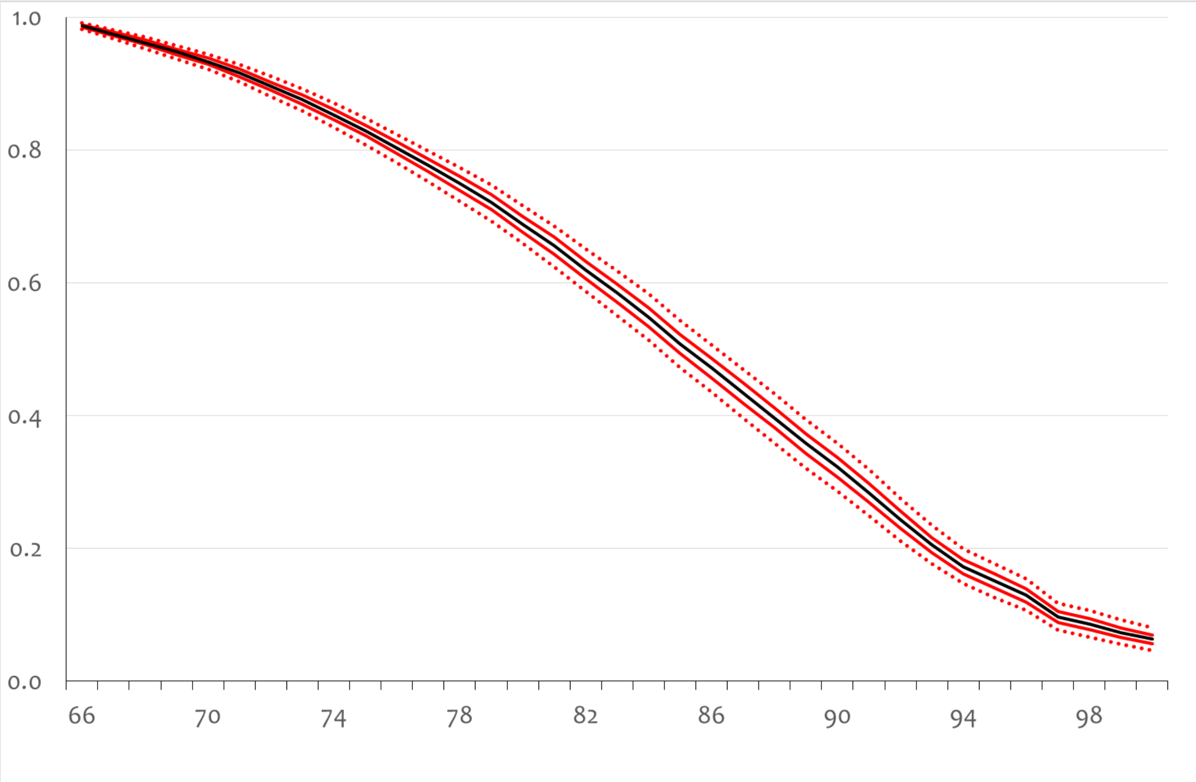
Figure 7: Estimated parameters from Lee-Carter Model





The figure shows that results of our baseline estimates of the Lee-Carter model, with the estimated alphas and betas being approximately linear in age, and the kappa following a stochastic trend over the years 1983-2000.

Figure 8: Fan chart of survival probabilities, male 65



This fan chart shows uncertainties surrounding the projections of survival probabilities, and this uncertainty is reflected by the shading in the fan charts. The central heavy line shows the most likely outcome (median), the two solid lines either side of the median show the 75th and 25th percentiles, and the dotted lines show the 95th and 5th percentiles.

Figure 9: Annuity Value Distributions, male 65

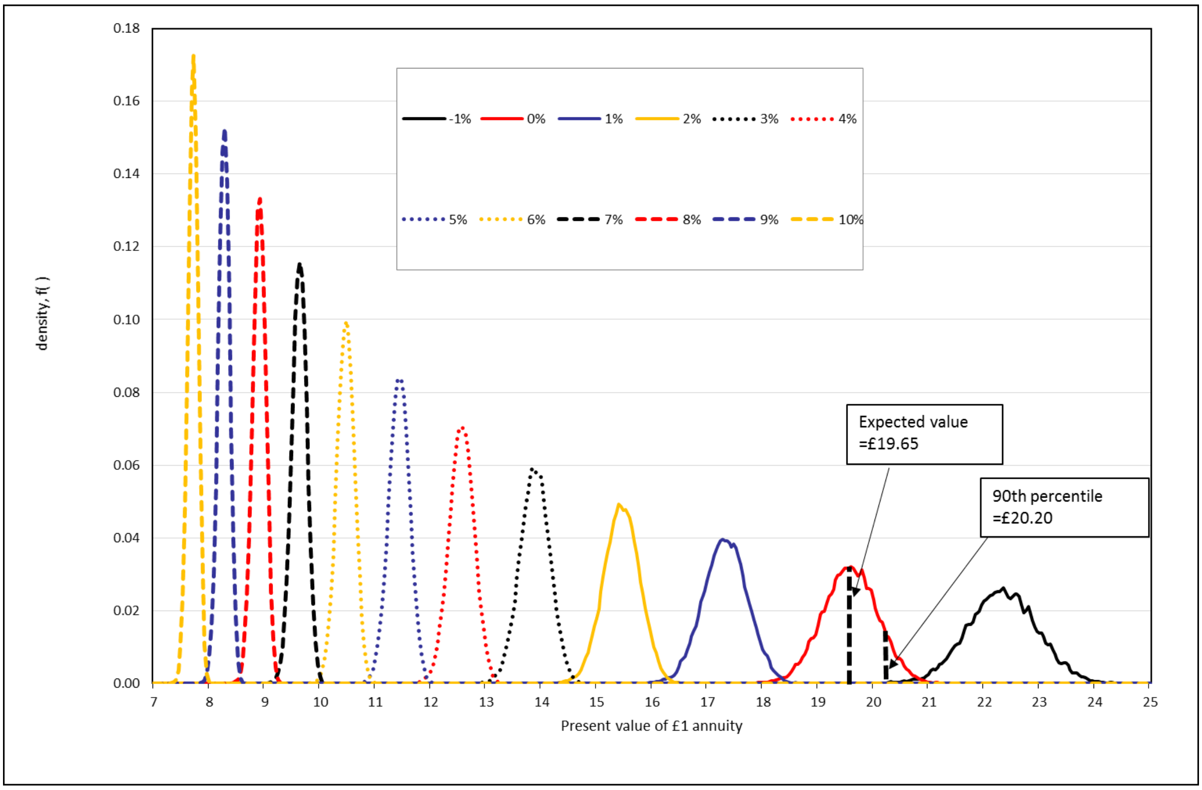


Figure illustrates (for different interest rates, ranging from -1% to 10%) the density functions of the present value of a £1 life annuity, based on the distribution of survival probabilities.

Figure 10: Money's worths using actual yields

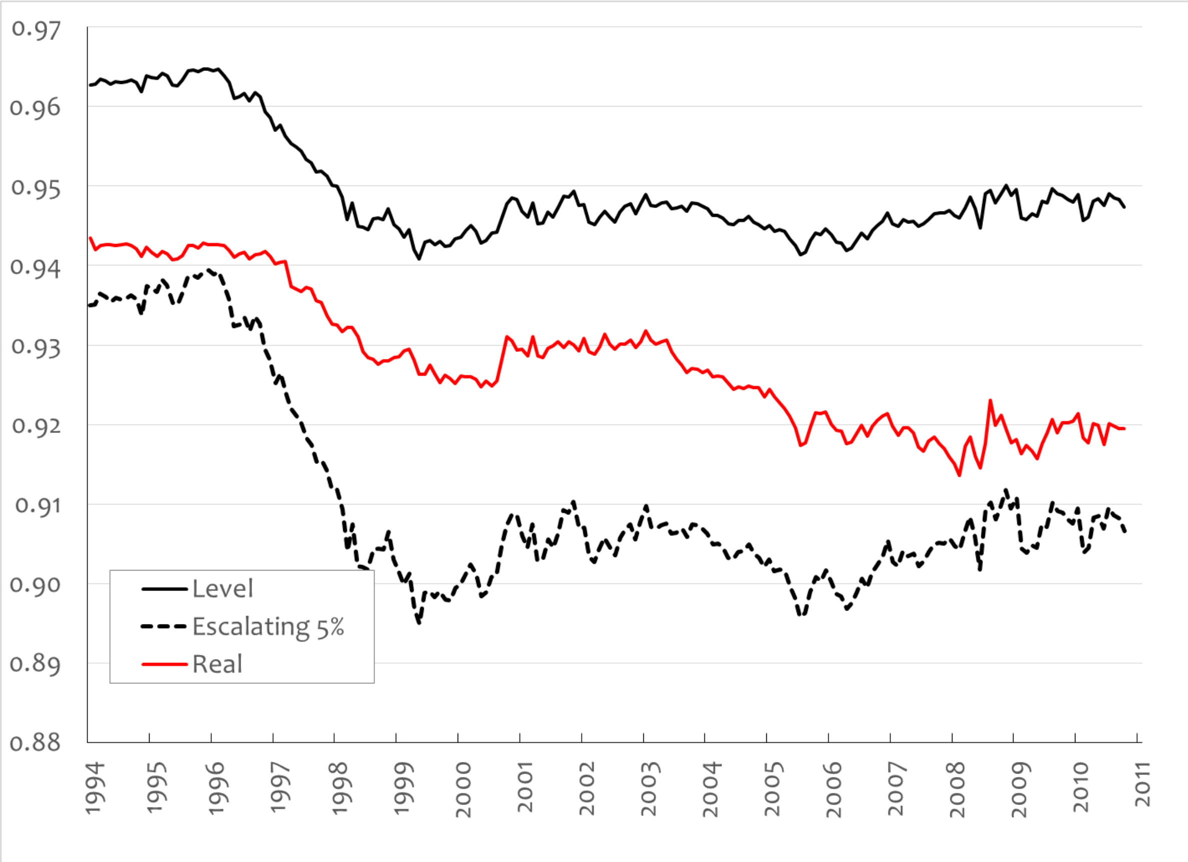


Figure shows time series of money's worths for level, real and escalating annuities based on an annuity provider pricing off the 90th centile of the annuity distribution.

Tables

Table 1: Summary of mortality assumptions in the FSA returns

Company	Mortality assumption
Aviva Life	88.5% of PCMA00
Canada Life	89% of RMV00 (plus further adjustments)
Hodge Life	65% of PCMA00
Legal and General	69.5% of PCMA00 (plus further adjustments)
Prudential	95% of PCMA00
Standard Life	88.4% of RMC00

The table reports the mortality assumptions and mortality tables used by the main annuity providers. Mortality table PCMA00 provides the mortalities of members of occupational defined-benefit pension schemes administered by life insurers; RMC00 and RMV00 summarise the mortality evidence of retirement annuity contracts for self-employed workers; RMV is for pensioners in receipt of a pension ("vested") and RMC is for both pensioners in receipt of a pension and for those still making contributions ("combined").

Table 2: Monthly Time Series Properties of Nominal Pension Annuity for 65-year old males and various alternative bond yields

	Annuity Rate for 65-year old males	Long-term: 10 year Government Bond Yield	Short-term: Bank of England Base rate	Interest rates on retail term deposits	Difference in Annuity Rate and Government Bond Yield
Panel A: Aug 1994 – April 2012					
Mean	7.96%	5.10%	4.43%	3.21%	2.86%
St.Dev.	1.70%	1.49%	2.09%	1.54%	
Correlation	0.93				
Panel B: Aug 1994 – July 2007					
Mean	8.54%	5.59%	5.34%	3.81%	2.95%
St. Dev.	1.63%	1.38%	1.07%	1.18%	
Correlation	0.92				
Panel C: Aug 2007 – Apr 2012					
Mean	6.40%	3.77%	1.88%	1.62%	2.63%
St. Dev.	0.49%	0.79%	2.12%	1.23%	
Correlation	0.88				

The table presents descriptive statistics on the monthly time series of average nominal annuity rates in the compulsory annuity market (CPA), long-term and short-term government bond yields and rates on retail term deposits, over the period 1994 to 2012 in Panel A, and for two sub-periods: 1994-2007 in Panel B, and 2007-2012 in Panel C. Annuity data provided by MoneyFacts and all bond data are taken from the Bank of England web-site.

Table 3: Monthly Time Series Properties of Real Pension Annuity for 65-year old males and various alternative bond yields

	RPI-linked Annuity Rate for 65-year old males	Long-term: 10 year Real Government Bond Yield	Difference in Real Annuity Rate and Real Government Bond Yield
Panel A: Sept 1998 – April 2012			
Mean	4.93%	1.60%	3.34%
St.Dev.	0.95%	0.77%	
Correlation	0.81		
Panel B: Sept 1998 – July 2007			
Mean	5.43%	2.02%	3.41%
St. Dev.	0.78%	0.35%	
Correlation	0.71		
Panel C: Aug 2007 – Apr 2012			
Mean	4.01%	0.80%	3.20%
St. Dev.	0.34%	0.73%	
Correlation	0.88		

The table presents descriptive statistics on the monthly time series of average real annuity rates in the compulsory annuity market (CPA) and real long-term government bond yields over the period 1994 to 2012 in Panel A, and for two sub-periods: 1994-2007 in Panel B, and 2007-2012 in Panel C.

Table 4: Testing for Differences in Money's worths by age, and product type

		1994.ix - 2000.xii: 80 Life-Table	t-test	1998.ix - 2003.xii: 92 Life-Table	t-test	2001.i - 2004.xii: medium cohort	t-test	2004.v - 2012.iv: oo Life-Table	t-test
Panel A: Different Ages									
Level, NG, male 60	Obs.	77		65		48		96	
	Mean	0.886	8.68***	0.926	5.54***	0.916	4.64*	0.864	6.55***
	St.dev	0.011		0.068		0.072		0.024	
Level, NG, male 65	Obs.	77		65		48		96	
	Mean	0.866	Base-case	0.909	Base-case	0.927	Base-case	0.859	Base-case
	St.dev	0.013		0.061		0.069		0.021	
Level, NG, male 70	Obs.	77		65		48		96	
	Mean	0.845	12.42***	0.889	6.15***	0.933	1.94*	0.854	4.16***
	St.dev	0.016		0.053		0.063		0.018	
Level, NG, male 75	Obs.	41		65		48		96	
	Mean	0.812	15.18***	0.872	6.05***	0.925	0.2	0.850	4.43***
	St.dev	0.014		0.046		0.052		0.017	
Panel B: Different Guarantees									
Level, 5-year guarantees, male 65	Obs.	77		65		48		96	
	Mean	0.881	29.00***	0.915	10.67***	0.932	6.39***	0.867	48.42***
	St.dev	0.014		0.059		0.067		0.021	
	Obs.	0		23		35		96	

Level, 10-year guarantees, male aged 65	Mean			0.847	-7.02***	0.893	-0.81	0.873	-17.80***
	St.dev			0.029		0.032		0.022	
Panel C: Different products									
Real (RPI-linked), NG, male 65	Obs.	28		64		48		96	
	Mean	0.784	18.61***	0.840	18.57***	0.867	16.11***	0.768	15.58***
	St.dev	0.007		0.063		0.064		0.027	
Escalating 5%, NG, male 65	Obs.	0		23		35		96	
	Mean			0.770	11.50***	0.856	7.05***	0.802	10.14***
	St.dev			0.042		0.048		0.033	

The table presents money's worth values for annuities by age, guarantee and product type (real and escalating), for four sub-periods corresponding to the relevant mortality tables (PML80 for data from 1994-2001; PML92 for data from 1999-2002; medium cohort for data from 2002-2005; and PNML00 for data from 2005 to 2012). The first row in Panel A reports the MW of the base case of a level annuity for male aged 65 with no guarantee (NG) The column "t-test" reports a t-test on the differences of matched pairs, i.e. compares the money's worth of the relevant annuity product with the base-case of the equivalent level annuities NG, male aged 65. The standard errors for these tests are Newey-West standard errors with 10 lags. Where *, **, *** denotes significance at 90, 95 and 99 per cent respectively.

Table 5: Stochastic Money's Worth Calculations

	Panel A: Money's worth of level annuities				Panel B: Money's worth of escalating 5% annuities				Panel C: Difference in money's worth	
Quantile:	0.5	0.9	0.95		0.5	0.9	0.95		0.9	0.95
Interest rate										
-1%	1.003	0.899	0.875		1.015	0.825	0.781		0.075	0.094
0%	1.002	0.910	0.890		1.011	0.843	0.803		0.069	0.086
1%	1.001	0.920	0.902		1.008	0.860	0.824		0.062	0.078
2%	1.001	0.929	0.913		1.006	0.875	0.843		0.055	0.071
3%	1.000	0.937	0.923		1.004	0.889	0.860		0.049	0.063
4%	1.000	0.943	0.931		1.003	0.901	0.876		0.044	0.055
5%	1.000	0.949	0.938		1.002	0.912	0.890		0.038	0.049
6%	1.000	0.954	0.944		1.001	0.921	0.902		0.034	0.043
7%	1.000	0.959	0.950		1.000	0.930	0.913		0.030	0.037
8%	0.999	0.963	0.955		1.000	0.937	0.922		0.026	0.033
9%	0.999	0.966	0.959		1.000	0.943	0.929		0.023	0.030
10%	0.999	0.969	0.963		1.000	0.949	0.936		0.021	0.026

Panels A and B of the table shows the money's worth of a £1 annuity (nominal and escalating 5%) at different interest rates, where MW is the ratio of the expected value of the annuity payments, relative to the relevant percentile of the annuity distribution (50th, 90th, and 95th percentile). The survival projections are made from the Lee-Carter model in equation (6) using ages 61-100. Panel C shows the difference in the respective numbers in the first two panels.

Table 6: Difference in money's worth calculations using alternative mortality models

	Lee-Carter												Cairns-Blake-Dowd		
	Least Squares						Maximum likelihood						Maximum likelihood		
Data: ages	60-100		61-100		65-100		60-100		61-100		65-100		60-100	61-100	65-100
Trend	S	D	S	D	S	D	S	D	S	D	S	D	S	S	S
-1%	0.119	0.022	0.075	0.020	0.089	0.025	0.040	0.018	0.077	0.027	0.077	0.028	0.013	0.008	0.010
0%	0.118	0.020	0.069	0.017	0.082	0.022	0.037	0.016	0.070	0.024	0.069	0.024	0.011	0.007	0.009
1%	0.113	0.018	0.062	0.015	0.073	0.019	0.034	0.015	0.062	0.021	0.062	0.021	0.010	0.007	0.008
2%	0.109	0.016	0.055	0.012	0.065	0.016	0.032	0.013	0.055	0.018	0.055	0.018	0.009	0.006	0.008
3%	0.104	0.014	0.049	0.011	0.057	0.014	0.029	0.012	0.048	0.016	0.048	0.016	0.008	0.006	0.007
4%	0.099	0.013	0.044	0.009	0.051	0.012	0.026	0.011	0.042	0.014	0.042	0.013	0.008	0.005	0.006
5%	0.092	0.011	0.038	0.008	0.044	0.011	0.024	0.009	0.037	0.012	0.037	0.012	0.007	0.005	0.006
6%	0.085	0.010	0.034	0.007	0.039	0.009	0.022	0.008	0.032	0.010	0.032	0.010	0.006	0.004	0.005
7%	0.079	0.009	0.030	0.006	0.035	0.008	0.020	0.008	0.028	0.009	0.028	0.009	0.006	0.004	0.005
8%	0.073	0.008	0.026	0.005	0.031	0.007	0.018	0.007	0.024	0.008	0.024	0.008	0.005	0.004	0.005
9%	0.067	0.007	0.023	0.005	0.027	0.006	0.016	0.006	0.021	0.007	0.021	0.007	0.005	0.003	0.004
10%	0.062	0.006	0.021	0.004	0.025	0.005	0.014	0.005	0.018	0.006	0.018	0.006	0.004	0.003	0.004

The tables show the difference between the money's worth for a level annuity and the money's worth for an escalating annuity, where each column reports the results based on a different mortality model (Lee-Carter or Cairns-Blake-Dowd), or a different sub-sample of the data. Projection is either via a stochastic trend (S) or a deterministic trend (D) and in all cases it is assumed that the life insurer prices annuities off the 90th centile. The third column (model estimated using ages 61-100 and projected with a stochastic trend) repeats the penultimate column of Table 5 Panel C.

Appendices

A.1 Alternative tests of differences in the money's worth

Table A1: Tests of differences in the log-money's worths, male aged 65, level, no-guarantee

	1994.ix - 2000.xii: 80 Life-Table	**1998.ix - 2003.xii: 92 Life-Table**	**2001.i - 2004.xii: medium cohort**	**2005.iii - 2012.iv: 00 Life-Table**
Panel A: Different ages (level annuities, no guarantee)				
Male 70	8.48*** (n = 77)	6.02*** (n = 65)	4.46*** (n = 48)	6.76*** (n = 87)
Male 70	-11.66*** (n = 77)	-6.72*** (n = 65)	2.06* (n = 48)	-4.23*** (n = 87)
Male 75	-14.15*** (n = 41)	-6.52*** (n = 65)	-0.08 (n = 48)	-4.51*** (n = 87)
Panel B: Different guarantees (level annuities, male 65)				
5-year guarantee	32.26*** (n = 77)	8.36*** (n = 65)	5.53*** (n = 48)	54.07*** (n = 87)
10-year guarantee		6.28*** (n = 23)	0.88 (n = 35)	18.23*** (n = 87)
Panel C: Different back-loading (no guarantee male 65)				
Real (RPI-linked)	-19.15*** (n = 28)	-15.50*** (n = 64)	-22.28*** (n = 48)	-15.01*** (n = 87)
Escalating 5%		-9.59*** (n = 23)	-6.66*** (n = 35)	-9.36*** (n = 87)

The table presents t-tests for the equality of money's worth values for different annuity types, analogous to Table 4, but instead of testing whether the money's worths are equal, it tests whether the log-money's worths are equal (again using a matched pairs test). In every case an annuity type is compared to the level annuity without guarantee for a male aged 65. The standard errors for these tests are Newey-West standard errors with 10 lags, but other lag lengths led to very similar results. Where *, **, *** denotes significance at 90, 95 and 99 per cent respectively.

A.2 Estimating the Lee-Carter model

In this section we explain our implementation of the model introduced by Lee and Carter (1992); proofs of the results and further exposition can be found in Girosi & King (2008, pp.34.ff) and Pitacco et al (2008, pp.169-173 & 186.ff.). We estimate equation (6) in the main text,

(A.1) $$ln(1 - p_{x,t}) = lnq_{x,t} = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \qquad \varepsilon_{x,t} \sim N(0, \sigma_x^2)$$

This specification does not completely identify the parameters, so identifying restrictions (which have no effect on the analysis) are used:

(A.2) $$\sum_t \kappa_t = 0, \qquad \sum_x \beta_x = 1.$$

We consider two ways to estimate the parameters. First, we use Least Squares, the original method proposed by Lee and Carter (1992) and still treated as the conventional way to estimate the model in Girosi and King (2008) and Pitacco *et al* (2008): if life insurers and actuaries were using a Lee-Carter model during the period 1994-2011, it is likely that they were using a Least Squares estimator.

A sample of the death-rate data that we use are shown in the table below:

	1983	1984	. . .	2000
60	0.013	0.015		0.011
61	0.010	0.007		0.010
62	0.012	0.019		0.010
63	0.019	0.014		0.009
64	0.034	0.021		0.018
65	0.021	0.021		0.012
. . .				
100	0.174	0.517		0.229

It can be seen that our data set for ages 60-100 can be arranged in a 41×18 matrix, more generally in an $X \times T$ matrix. We denote the logarithm of this matrix as

(A.3) $$\mathbf{Q} = (lnq_{x,t}) \in \mathbb{R}^X \times \mathbb{R}^T$$

(nb ages in rows, years in columns). The least-squares estimation of the intercept term is straight-forward and intuitive: given the constraint that $\sum_t \kappa_t = 0$, just take the row means to get

(A.4) $$\hat{\alpha}_x = \frac{\sum_{t=1983}^{2000} \ln q_{x,t}}{18}$$

We stack the estimates of the alphas into an $X \times 1$ vector $\hat{\boldsymbol{\alpha}}$ and obtain the de-(row)meaned data

(A.5) $$\widetilde{\mathbf{Q}} \equiv \mathbf{Q} - \hat{\boldsymbol{\alpha}}\mathbf{1}$$

where **1** is a row vector of ones. We estimate the uncertainty in our values of alpha(hat) using the conventional standard errors of the means.

To estimate the betas and kappas, note that, using the singular-value-decomposition theorem, the $X \times T$ matrix $\widetilde{\mathbf{Q}}$ can be written as

(A.6) $$\widetilde{\mathbf{Q}} = \mathbf{BLK}$$

where **L** is a diagonal matrix with the singular values put in descending order and the vector **B** contains the principal components. The estimates of β_x are just the first column of the matrix **B** and the estimates of κ_t are just the first row of **K**. To estimate the uncertainty in our values of beta(hat) using the bootstrap procedure suggested in the appendix of Lee and Carter: denote the residuals as

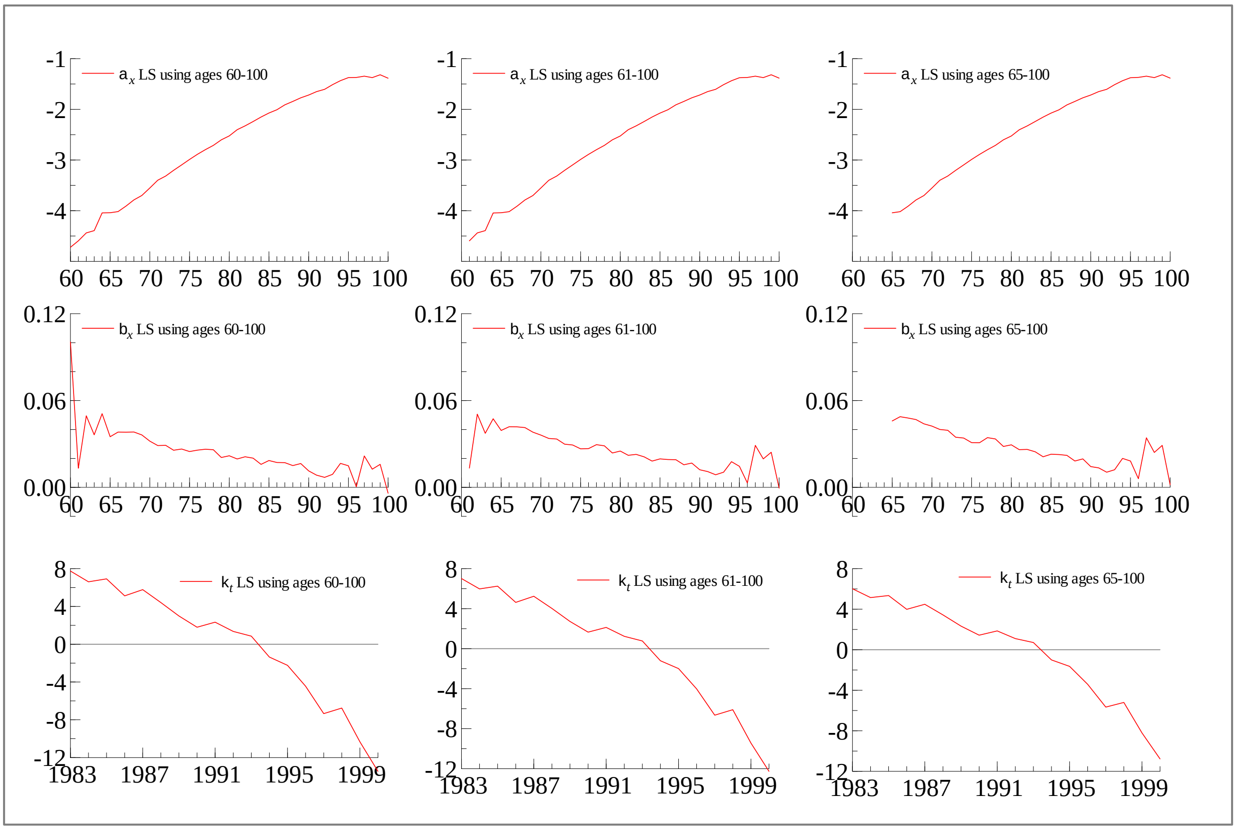
(A.7) $$\hat{\varepsilon}_{x,t} = lnq_{x,t} - \hat{\alpha}_x - \hat{\beta}_x \hat{\kappa}_t$$

then repeatedly re-sample with replacement residuals and add them to the fitted value of **Q**. The resulting data set can be used to generate new values of beta(hat). We use 500 bootstrap replications to calculate the covariance matrix of the betas. As in Lee and Carter (1992) we assume that the betas are independent of the alphas.

The only remaining issue is how many of the data to use. Many of the death rates for ages below 60 are zero, so it is impossible to take logs: there is one observation for which the death rate for age 60 is zero. For this reason we consider three sub-samples of the data: ages 60-100 (where we replace the zero death by half a death); ages 61-100 (the largest data set with no zero death rates) and ages 65-100 (the smallest data set which still enables us to estimate an annuity for a 65-year old).

A comparison of the Lee-Carter parameters is shown in the Figure A1 below:

Figure A1: Estimates of the Lee-Carter parameters using different sub-samples of the data.



The figure shows that results of Lee-Carter model parameters (alpha, beta, kappa) for different sub-samples of the data.

An alternative estimation procedure would be to use maximum likelihood, which is possible for us since we have data on both the exposed-to-risk (i.e. number of individuals facing the hazard of death) and the number of deaths. We now amend equation (A.1) so that the death probability is written

(A.8) $$q_{x,t} = exp\{\alpha_x + \beta_x \kappa_t\}$$

The likelihood is

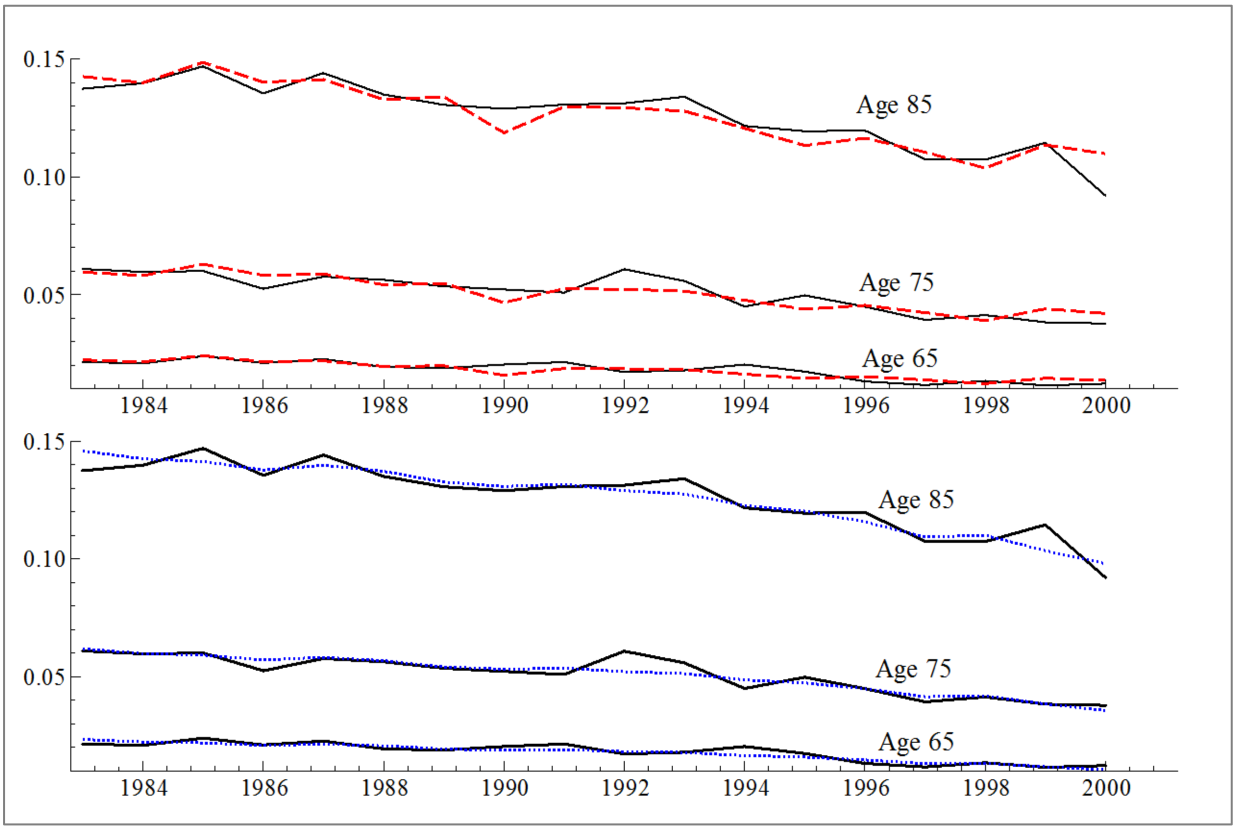
(A.9) $$\prod_t \prod_x C(N_{x,t}, D_{x,t}) q_{x,t}^{D(x,t)} (1 - q_{x,t})^{N(x,t)-D(x,t)}$$

where $C(N_{x,t}, D_{x,t})$ iis the combinatorial function, $N_{x,t}$ is the number of lives (exposed to risk) and $D_{x,t}$ is the number of deaths. The log-likelihood (ignoring the constant) is

(A.10) $$\sum_t \sum_x D_{x,t}(\alpha_x + \beta_x \kappa_t) + (N_{x,t} - D_{x,t}) ln(1 - exp\{\alpha_x + \beta_x \kappa_t\})$$

which can be maximised subject to the identifying constraints in (A.2). As with our Least Squares estimates estimated the covariances of the alphas and betas by bootstrapping. To get some idea of the goodness of fit, we plot actual and fitted death rates for models estimated by both Least Squares and Maximum Likelihood in the figure below:

Figure A2: Actual and fitted death rates from the Lee-Carter model



The figure plots actual (one-year) death rates (solid black line) for selected ages and corresponding fitted death rates from the Lee-Carter model estimated by Least Squares (top panel, dashed red line) and Maximum Likelihood (bottom panel, dotted blue line).

The final issue is the dynamic model of the kappas to allow projection into the future.

We used two models: a unit root process (as proposed by Lee and Carter, 1992, and widely used in the literature) and a deterministic trend as a robustness check:

(A.11) $$\kappa_t = \lambda + \kappa_{t-1} + \psi_t \qquad \text{or} \qquad \kappa_t = \hat{\kappa}_{2000} + \lambda t + \psi_t$$

We use our parameter estimates $\hat{\alpha}_x$, $\hat{\beta}_x$, $\hat{\sigma}_\varepsilon^2$, $\hat{\lambda}$ and $\hat{\sigma}_\psi^2$, together with $\hat{\kappa}_{2000}$ to make projections about future death rates and to quantify the risk in those projections using Monte Carlo simulations: in each case we simulate 10,000 replications which allows us to create the distribution of the relevant random variables. Within our model, projections are stochastic for three reasons. First, we must project the values of the κ_t into the future (the randomness arising from the random draws of ψ_t). Second, the actual log-mortality deviates from the expected projection due to the error term ε_{xt}. A third source of uncertainty arises because we do not know the true values of α_x, β_x and λ, although (as also noted by Lee and Carter, 1992), the uncertainty created in the estimation of α_x and β_x is not quantitatively important. In each replication of the Monte Carlo, we first model the parameter uncertainty by generating values of α_x, β_x and λ by drawing from

$$\tilde{\alpha}_x \sim \mathcal{N}\left(\hat{\alpha}_x, \operatorname{var}[\hat{\alpha}_x]\right)$$
$$\tilde{\beta}_x \sim \mathcal{N}\left(\hat{\beta}_x, \operatorname{var}\left[\hat{\beta}_x\right]\right)$$
$$\tilde{\lambda} \sim \mathcal{N}\left(\hat{\lambda}, \operatorname{var}\left[\hat{\lambda}\right]\right)$$

where the variances are calculated by the crude standard errors of the least-squares estimation; second, we use the random draw of $\tilde{\lambda}$ together with the parameter estimate of $\hat{\sigma}_\psi^2$ to generate the future values of the kappas using either

$$\tilde{\kappa}_{2000+t} = \tilde{\lambda} + \tilde{\kappa}_{2000+t-1} + \textstyle\sum_{i=1}^{t} \tilde{\psi}_i \qquad \text{or} \qquad \tilde{\kappa}_{2000+t} = \hat{\kappa}_{2000} + \tilde{\lambda} t + \textstyle\sum_{i=1}^{t} \tilde{\psi}_i$$

where $\tilde{\psi}_t \sim \mathcal{N}\left(0, \hat{\sigma}_\psi^2\right)$; and finally we calculate the log-death probabilities using

$$\ln q_{x,2000+t} = \tilde{\alpha}_x + \tilde{\beta}_x \tilde{\kappa}_{2000+t} + \tilde{\varepsilon}_{x,t}$$

where the deviations of death probabilities from the forecast are drawn from a random process of the form $\tilde{\varepsilon}_{x,t} \sim \mathcal{N}\left(0,\hat{\sigma}_{\varepsilon}^{2}\right)$. Notice that we take the values of $\hat{\sigma}_{\varepsilon}^{2}$, $\hat{\sigma}_{\psi}^{2}$ as given. The procedure just outlined generates a complete set of log death probabilities for all ages for the relevant period which can then be used to generate the survival probabilities and the corresponding value of an annuity and the Value-at-Risk. We repeat the entire process 10,000 times. Figures 8, 9 and 10 and Tables 5 and 6 are all based on the probability distribution of death probabilities from this Monte Carlo procedure.

In the proof to Proposition 2, that the money's worth will be lower for an escalating annuity than a level annuity, we make use of the "concordance ratio" i.e. the ratio of the expected survival probability to the survival probability at the upper confidence interval. The sufficient condition for our result requires this ratio to be falling (uncertainty increasing) with the time horizon. A sample concordance ratio from our models that demonstrates this property is illustrated in Figure A3 below:

Figure A3. Concordance ratio

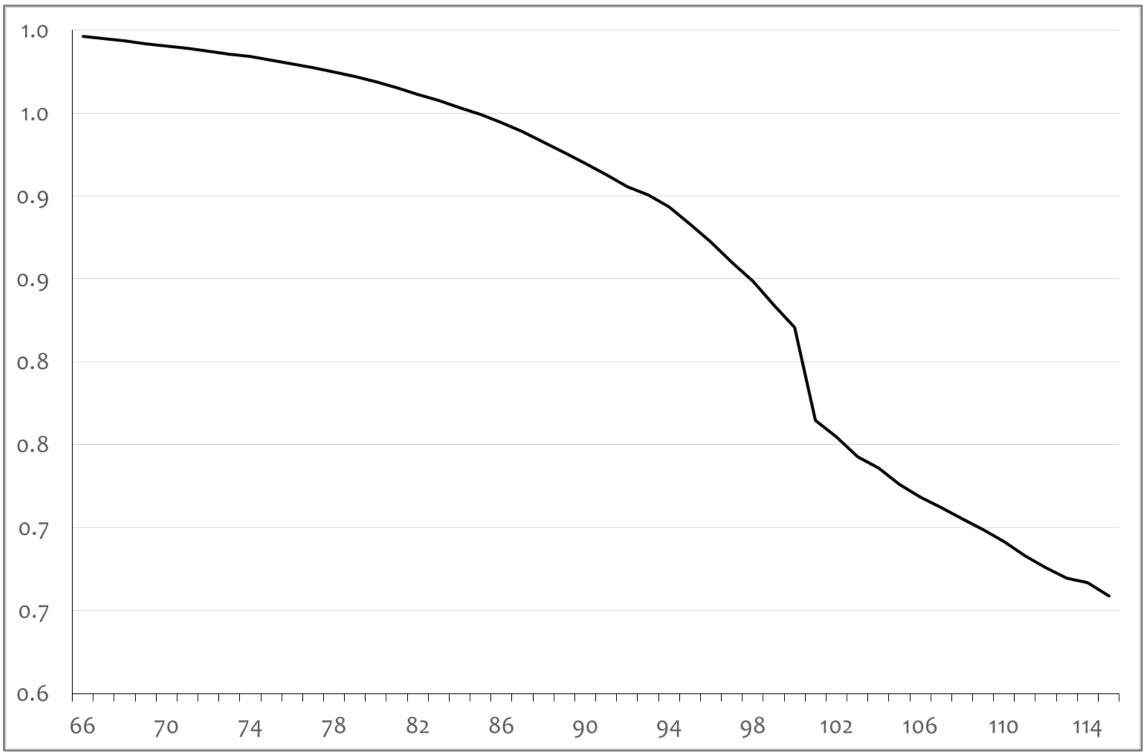


Figure shows an example of the concordance ratio estimated from the baseline Lee-Carter model, Least Squares, ages 61-100.

A3. Estimating the Cairns-Blake-Dowd model

The Cairns-Blake-Dowd model is based on the relationship

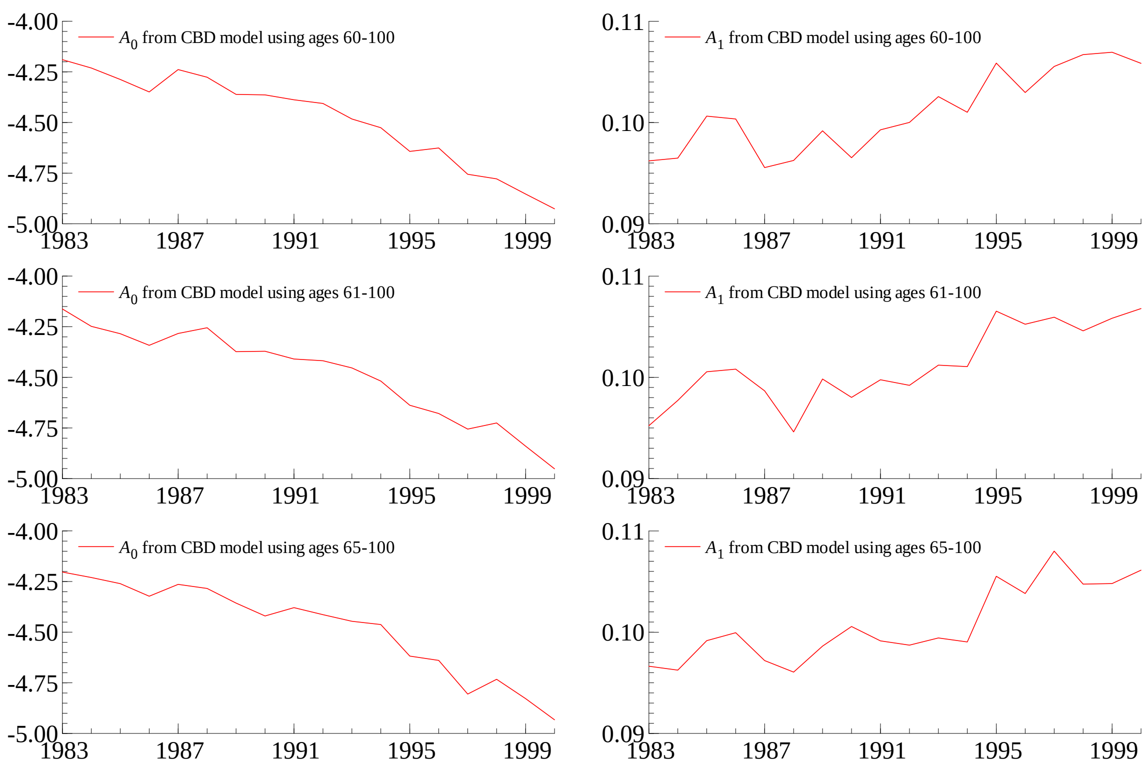
(A.12) $$ln\left(\frac{q_{t,x}}{1-q_{t,x}}\right) = A_0(t) + A_1(t)x$$

where the two unobserved factors are modelled as unit root processes:

(A.13) $$\begin{pmatrix} A_0(t) \\ A_1(t) \end{pmatrix} = \begin{pmatrix} \mu_0 \\ \mu_1 \end{pmatrix} + \begin{pmatrix} A_0(t-1) \\ A_1(t-1) \end{pmatrix} + \begin{pmatrix} \varepsilon_0(t) \\ \varepsilon_1(t) \end{pmatrix}$$

In the original model (Cairns, Blake and Dowd, 2006) the first equation was estimated by Least Squares, but here we consider only Maximum Likelihood estimation to obtain estimates of the dynamic factors $A_0(t)$ and $A_1(t)$. The dynamic process is estimated in a second-stage regression by Least Squares. Our estimates of the dynamic factors (strictly speaking $\hat{A}_0(t)$ and $\hat{A}_1(t)$) are illustrated below.

Figure A4: Estimated coefficients from Cairns-Blake-Dowd model



The figures plot the estimated parameters from the Cairns-Blake-Dowd model for the three sub-samples.

A4. Differences between models and model uncertainty

In the main text, our analysis looks at how much money's worths would be affected if both the life insurer and the research were using the same model and the only difference was that the researcher was calculating the expected value of the annuity and the life insurer was valuing the annuity off the upper confidence interval. In practice there must also be uncertainty about which model to use. This could affect the money's worths in two distinct ways. First, the researcher and the life insurer could use different models (but the life insurer still price the annuity off the confidence interval generated by a single model). Second, the life insurer could build an additional margin into the price to allow for the model uncertainty.

We do not attempt to quantify formally the model uncertainty. But a useful check is to see how much the expected annuity values differ from different models and estimation techniques. Table A2 reports the expected value of the annuity from the fifteen models that we have estimated.

There are two very notable features in this table. First, the Lee-Carter figures are all very different (and much higher) than the CBD figures. So if an economist valued the money's worth using the CBD model (which gives a low annuity value suggesting a high annuity rate) and life insurers used the Lee-Carter model then the money's worth would appear to be low (regardless of any issue of uncertainty). With an interest rate of 5 per cent, the average annuity value from the average Lee-Carter model is 11.71 and from the CBD model it is 11.05, suggesting a money's worth of 0.94 for level annuities: for escalating annuities the corresponding figures are 20.56, 18.31 and a money's worth of 0.89.

The second issue is that while the CBD models all provide very similar estimates regardless of the data sub-sample used, the Lee-Carter model is sensitive to all three variations that we consider: sub-sample used for estimation, estimation technique and type of trend used for the projection. When the interest rate is 5 per cent, the coefficient of variation of the twelve LC estimates is 0.029 for level annuities and 0.043 for escalating annuities. The effect of model uncertainty is large.

Table A2, Panel A: Expected annuity values from different models: level annuities

	Lee-Carter												Cairns-Blake-Dowd		
	Least Squares						Maximum likelihood						Maximum likelihood		
Data: ages	60-100		61-100		65-100		60-100		61-100		65-100		60-100	61-100	65-100
Trend	S	D	S	D	S	D	S	D	S	D	S	D	S	S	S
-1%	21.1	22.3	23.6	23.4	24.8	23.5	24.5	23.4	24.9	23.6	25.0	23.6	20.6	20.7	20.6
0%	18.5	19.6	20.5	20.4	21.4	20.5	21.3	20.5	21.5	20.5	21.6	20.5	18.3	18.3	18.3
1%	16.3	17.3	18.1	18.0	18.7	18.0	18.7	18.0	18.8	18.0	18.8	18.0	16.3	16.4	16.3
2%	14.6	15.4	16.0	16.0	16.5	16.0	16.6	16.0	16.6	16.0	16.6	16.0	14.7	14.7	14.7
3%	13.1	13.9	14.3	14.3	14.7	14.3	14.8	14.4	14.8	14.3	14.8	14.3	13.3	13.3	13.3
4%	11.9	12.5	12.9	12.9	13.2	12.9	13.3	13.0	13.3	12.9	13.3	12.9	12.1	12.1	12.1
5%	10.8	11.4	11.7	11.7	12.0	11.7	12.1	11.8	12.0	11.7	12.0	11.7	11.0	11.1	11.1
6%	9.9	10.4	10.7	10.7	10.9	10.7	11.0	10.7	10.9	10.7	10.9	10.7	10.1	10.2	10.2
7%	9.1	9.6	9.8	9.8	10.0	9.8	10.1	9.9	10.0	9.8	10.0	9.8	9.4	9.4	9.4
8%	8.5	8.9	9.1	9.0	9.2	9.0	9.3	9.1	9.2	9.0	9.2	9.0	8.7	8.7	8.7
9%	7.9	8.2	8.4	8.4	8.5	8.4	8.6	8.4	8.5	8.4	8.5	8.4	8.1	8.1	8.1
10%	7.3	7.7	7.8	7.8	7.9	7.8	8.0	7.8	7.9	7.8	7.9	7.8	7.5	7.6	7.6

Figures show the expected annuity values for a level annuity, where each column reports the results based on a different mortality model or a different sub-sample of the data and projection is either via a stochastic trend (S) or a deterministic trend (D). Each column corresponds to a different model: 15 in total.

Table A2, Panel B: Expected annuity values from different models: escalating annuities

	Lee-Carter												Cairns-Blake-Dowd		
	Least Squares						Maximum likelihood						Maximum likelihood		
Data: ages	60-100		61-100		65-100		60-100		61-100		65-100		60-100	61-100	65-100
Trend	S	D	S	D	S	D	S	D	S	D	S	D	S	S	S
-1%	46.9	47.8	53.8	52.9	59.3	53.5	54.8	51.1	60.2	54.2	60.5	54.2	40.9	40.7	40.7
0%	38.9	40.1	44.4	43.8	48.3	44.1	45.5	42.7	48.9	44.6	49.1	44.6	35.0	34.9	34.9
1%	32.7	34.1	37.1	36.7	40.0	37.0	38.3	36.1	40.3	37.3	40.5	37.2	30.3	30.2	30.2
2%	27.9	29.2	31.5	31.2	33.6	31.3	32.6	30.9	33.8	31.5	34.0	31.5	26.4	26.4	26.3
3%	24.0	25.3	27.0	26.8	28.6	26.9	28.0	26.7	28.7	27.0	28.8	27.0	23.2	23.2	23.2
4%	21.0	22.2	23.4	23.3	24.6	23.3	24.3	23.3	24.7	23.4	24.8	23.4	20.5	20.6	20.5
5%	18.5	19.6	20.5	20.4	21.4	20.5	21.3	20.5	21.5	20.5	21.6	20.5	18.3	18.3	18.3
6%	16.4	17.4	18.2	18.1	18.8	18.1	18.8	18.1	18.9	18.1	19.0	18.1	16.4	16.5	16.4
7%	14.7	15.6	16.2	16.1	16.7	16.1	16.8	16.2	16.8	16.2	16.8	16.2	14.8	14.9	14.8
8%	13.3	14.1	14.5	14.5	15.0	14.5	15.0	14.6	15.0	14.5	15.0	14.5	13.5	13.5	13.5
9%	12.1	12.8	13.2	13.1	13.5	13.1	13.6	13.2	13.5	13.1	13.6	13.1	12.3	12.3	12.3
10%	11.0	11.7	12.0	12.0	12.3	11.9	12.3	12.0	12.3	12.0	12.3	12.0	11.3	11.3	11.3

Figures show the expected annuity values for an escalating annuity, where each column reports the results based on a different mortality model or a different sub-sample of the data and projection is either via a stochastic trend (S) or a deterministic trend (D). Each column corresponds to a different model: 15 in total.

A.5 Idiosyncratic mortality risk

We also consider the possibility that there is further risk to the life insurer because the life insurer has sold only a relatively small number of annuities. Table A.1 shows the number of annuities sold by leading life insurers in the UK taken from the FSA returns, which require life insurers to distinguish sales of nominal and real annuities: clearly the proportion of real annuities sold is tiny.

Table A3: Purchases of different annuity types, 2011

Company	Nominal		Real	
	No of purchases	Average purchase	No. of purchases	Average purchase
Aviva Annuities	58,692	£32,696	1521	£33,707
Canada Life	13,440	£32,696	909	£72,939
Hodge Life	859	£31,029	none	
Legal & General	25,928	£25,102	845	£18,073
Prudential	37,006	£25,653	307	£33,707
Standard Life	20,361	£14,844	245	£20,804

Number of nominal and real annuity contracts sold in UK compulsory purchase market, and average value of annuity contract by annuity provider. Source: various FSA Returns, Appendix 9.3, Form 47, rows 400 and 905.

As a consequence, sales of real annuities suffer not only from cohort mortality risk but also idiosyncratic mortality risk. We modelled this by generating distributions of annuity values for a group of 300 annuitants with the same cohort mortality risk for the underlying probabilities. For each replication (which had a different set of cohort mortality risks), the number of annuitants that survived was drawn from a Binomial probability distribution.

Using the baseline Lee-Carter LS model with stochastic projection estimated from the sub-sample 61-100, the resulting figures for the money's worths are reported in the table below. Even for low interest rates the additional risk is small.

Table A4: Difference in money's worth calculations incorporating idiosyncratic mortality risk

	Large sample		300 Policies	
	90th	**95th**	**90th**	**95th**
-1%	0.075	0.094	0.076	0.100
0%	0.069	0.086	0.069	0.090
1%	0.062	0.078	0.062	0.081
2%	0.055	0.071	0.055	0.072
3%	0.049	0.063	0.049	0.063
4%	0.044	0.055	0.043	0.055
5%	0.038	0.049	0.039	0.049
6%	0.034	0.043	0.035	0.043
7%	0.030	0.037	0.031	0.037
8%	0.026	0.033	0.028	0.033
9%	0.023	0.030	0.024	0.030
10%	0.021	0.026	0.021	0.027

Figures show the difference between MW for level and escalating annuities assuming pricing at the 90^{th} or 95^{th} percentile. The first two columns repeat the final two columns of Table 5 Panel C. The last two columns show the additional difference in the MW from incorporating idiosyncratic mortality risk.